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The New Form of a Mathematical Model of a Cable suspended Parallel Robot Caused by Using of a New Winch Form

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Abstract

This paper presents novel results, which were initially developed based on wide analysis of well-known winch constructions, which were used as main parts of actuating subsystems of Cable suspended Parallel Robots (CPR) systems. The motivation of this work was the need to define and recognize dynamic characteristics of the winch such as radius and length of cable winding/unwinding, since it was shown that these variables highly affect dynamic response of CPR system. It has been proven that "jumpy" changes of these two variables generate oscillatory and unstable behavior of the system. It gets even more complicated when the CPR system has more of these winches and coupling of their dynamics occurs. If cable winding/unwinding radius and length have "jumpy" behavior, there is no control algorithm would stabilize the system. Un-modeled dynamics can highly affect oscillatory behavior and instability of the system. Because of these reasons, the new construction of the Cable suspended Parallel Robot that is using winches for performing single - row Radial Multi-layered smooth cable winding/unwinding (CPRRM system), and its mathematical model was defined and presented. Generally, it can be said that the analysis of the geometry of the selected mechanism is the first step, which is a very important step to establish the structural stability of the CPR systems. In order to discover all characteristics, geometry analysis of all system's components has to be first done, which as result, implies the possibility of the realization of the desired camera motion. The next step is to define a new form of kinematic and dynamic model. Then novel software package CPRRMSOFT (in MATLAB) was generated. This program package was used for the validation of the theoretical contributions. Also, it was used to check and choose all the parameters of the system and parameters of its components, and possible selection of the control structure. Validity of the obtained theoretical contributions has been illustrated in this paper through one case study. The whole procedure is iterative and repeatable. This is the procedure which leads toward implementation of complex CPR system.

Keywords: *Winding/Unwinding; Winch, Kinematics; Dynamics; Coupling.*

Nomenclature

DOF	degree of freedom	*lw*	cable length (m)
CWU	Cable Winding/Unwinding	γ	inclination of the cable respect to y_w (rad)
CPR	Cable suspended Parallel Robot	θ	rotation angle of the winch (rad)
R	radius of cable winding/unwinding (m)	G_{vi}	motor inertia characteristic
*	mathematical operation of multiplication	L_{vi}	motor damping characteristic
$\dot{}$	first derivative of any magnitude	S_{vi}	motor geometric characteristic

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u_i	voltage (V)
R_{ri}	rotor circuit resistance (Ω)
C_{Ei}	back electromotive force constant (V/(rad/s))
C_{Mi}	constant of the moment proportionality (Nm/A)
B_{Ci}	coefficient of viscous friction (Nm/(rad/s))
J_{ri}	moment of inertia (rotor and gear box (kgm^2)

1. Introduction

In the late 1980s and at the beginning of 1990s, the researchers and engineers started to work on a new type of robotic mechanism - Cable suspended parallel robot, CPR system. The first concepts of CPR systems have appeared in the United States of America and in Japan. With new types of cranes and parallel manipulators the new subarea of robotics began to emerge, and these systems became more and more popular in the upcoming years, and one of those systems was presented by Bruckmann and Pott [1].

One of the first systems driven by cables was constructed by Albus et al. [2], more than three decades ago and it was the NIST robo - crane. The NIST robo - crane was designed based on the Stewart platform parallel link manipulator and it provides a precise six degrees of freedom (DOF) control of its payload. This robotic mechanism presents one of the most known cable driven parallel systems together with the FALCON robot (Fast Load Conveyance) published by Kawamura et al. [3]. The Kawamura et al. [3] have described the development of an ultrahigh speed FALCON robot based on a cable driven parallel manipulation system.

CPR systems are very complex mechanisms. Their complexity comes from the need to specify a number of design parameters, like construction, number of cables, number of suspension points, load weight, etc. Actuators of these systems are their elementary parts and they usually consist of the motor, gear, and winch. Dynamic characteristics of each actuator are important for dynamic response of any CPR system. Also, the number of actuators affects the CPR system complexity.

In the previously published literature one type of winch was used, and one of the papers which described this type of winch was published by Von Zietzvitz et al. [4]; they have covered winches that have rotating and translational motion. Also, these winches perform single-layered cable winding/unwinding tasks. This constructive solution of the winch has also been used by Filipovic at al. [5 and 6]. The cable driven machines usually have at least two or more cable winding/unwinding subsystems. The RSCPR system has three cable winding/unwinding subsystems as presented by Filipovic at al. [5]. The RSCPR system was designed to use the winches described by Von Zietzvitz et al. [4] and in that case the following conditions were satisfied: $R_i = const, lw_i = const, \gamma_i = const.$ Design of this winch is characterized either by (a) two motors which generate rotation and translation of this winch in a coordinated fashion or (b) one motor and two gears. In the case of version (b), one gear rotates the winch, while the other gear moves it translatory based on its rotation. This type of winch is expensive and because of that application of the standard winch type for single - row radial multi-layered cable winding/unwinding has been analyzed by Kevac at al. [7]. During this analysis it was concluded that this construction generates abrupt changes of its characteristic variables. This was the reason for exclusion of this winch from further analysis. Filipovic and Kevac [8] are concentrated on design of a novel and simpler construction of the winch for performing single - row radial multi-layered smooth cable winding/unwinding. See Figure 1.

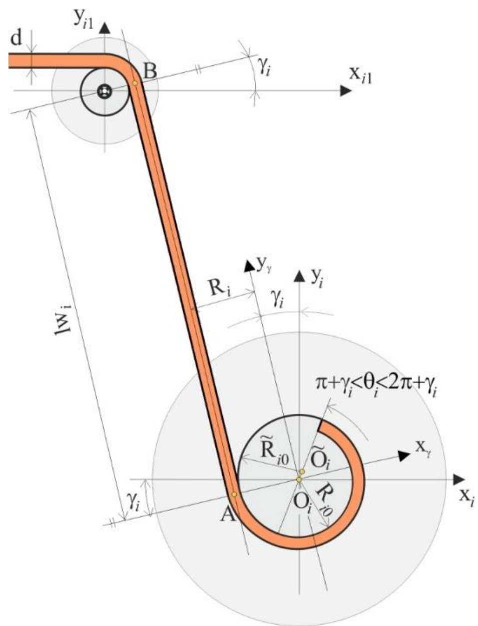

Figure 1. The positions of the smooth winding/unwinding system for $\pi + \gamma_i < \theta_i < 2\pi + \gamma_i$.

The mathematical model of cable winding/unwinding system, includes one actuator system (motor, gear and winch), is defined for several different constructions defined by Kevac and Filipovic [9]. The novelty of this mathematical model is inclusion of influence of change of winding/unwinding radius and cable length.

Kevac and Filipovic [9] described the new design of the winch for single - row radial multi-layered smooth cable winding/unwinding system intended for use as an integral subsystem of the new construction of the complex CPR system. By selecting this constructive solution of the winch, it is expected to achieve good CPR system's response without constructively caused discontinuities that could lead to instability and significant oscillations in the response of the system. Besides its application for CPR systems, the new design of the winch can be used for other type of cable driven systems.

In this paper, in first presents a novel design and the corresponding mathematical model of the CPR system. The novel winches for performing single - row radial multi-layered smooth cable winding/unwinding are used. This novel CPR construction is named CPRRM as an abbreviation from: Cable suspended Parallel Robot using winches for performing single - row Radial Multi-layered smooth cable winding/unwinding. CPRRM system includes three actuator's system (motor, gear and winch). For the verification of the defined mathematical model of the CPRRM system, the novel program packet CPRRMSOFT was used.

After the Introduction, in Section 2, the new constructive solution of the winch for smooth cable winding/unwinding as a part of the CPRRM system is described mathematically. In Subsection 2.1 kinematic model of the newly designed CPRRM system is formulated, and its dynamic model is defined in Subsection 2.2. By using the new mathematical model, the program package CPRRMSOFT is presented in Section 3. This program package is used to perform simulation experiments of the motion of the new CPRRM system's camera carrier in Section 4. In the last part of the paper the conclusions and observations are presented.

2. The Kinematic and Dynamic Models of the Novel CPRRM System

To avoid the so far used, well known, solution of the winch for the rotary and translatory motion by applying one-layered cable winding/unwinding process described by Von Zietzvitz et al. [4], as well as the known standard circular winch for performing single - row radial multi-layered cable winding/unwinding, modeled by Kevac at al. [7] their shortcomings were pointed out. By using these winches, several problems have been identified. This phenomenon has adverse effects on the system's dynamic response, and it causes instability and oscillations of the system. This structural instability of the standard winch has inspired the authors to design a new form of the winch for performing the process of cable winding/unwinding.

The constructive solution which does not cause abrupt changes in the cable velocity $\dot{lw_i}$ is required. The new constructive solution of the winch intended for performing smooth cable winding/unwinding system named as two - cylinder winch is developed by Filipovic and Kevac [8]. The idea of this kind of solution presented on Figure 1, emerged from the need to avoid the constructively generated unstable and oscillatory behavior of CPR system's dynamic responses caused by wrong selection of winch for cable winding/unwinding. New constructive winch solution can be defined as two joint semicylindrical bodies of different radii and it is presented in Figure 1. Because of the characteristics of this winch, it has been named: two - cylinder winch. The novel construction of the CPRRM system, Cable suspended Parallel Robot constructed using three winches for performing single - row Radial Multi-layered smooth cable winding/unwinding, is presented on Figure 2. Each winch performs smooth cable winding/unwinding; significant dynamic characteristics are $R_i,\ lw_i, \gamma_i$.

In this Section, the kinematic and dynamic models of the novel CPRRM construction, presented in Figure 2, are defined.

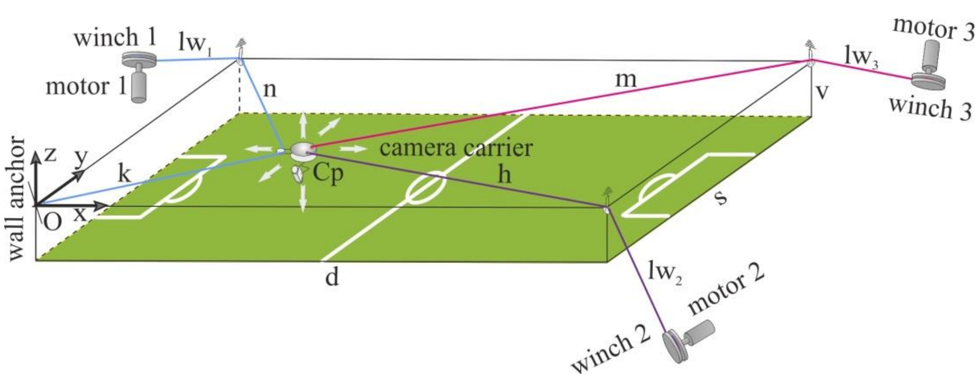

Figure 2. The CPRRM system in 3D space.

The similarity of this system to the RSCPR system developed by Filipovic at al. [5] is in the number of cables and in the way how camera carrying system is hanged in system's workspace. Both systems, the RSCPR and the CPRRM system, consist of three subsystems for performing cable winding/unwinding. However, the only difference is that the CPRRM system uses a new, constructive solution of winch for the winding/unwinding rope.

The new construction of the CPRRM system uses the winch for smooth cable winding/unwinding, shown in Figure 1. Kinematic and dynamic models of the CPRRM system with the new construction of winch will be developed in the next subsections.

2.1. The kinematic model of the CPRRM system

It has been presumed that position of the CPRRM system's camera carrier, defined in the Cartesian coordinate system, is labeled as $p = [x\ y\ z]^T$. These coordinates are named external coordinates. The winding/unwinding angle of each of the winches is defined by coordinate θ_i (i=1, 2, 3). It is also presumed that values of the winding/unwinding angle of all three winches are defined as internal coordinates, i.e. $\phi = [\theta_1\ \theta_2\ \theta_3]^T$.

In Figure 2, the workspace of the CPRRM system has been defined; the basis of the coordinate system is positioned at one of the upper corners of the workspace (point *O*); the overall dimensions of the workspace are defined as: d - length, s - width, v - height. Geometrically, based on Figure 2, one can define the following equations which describe hanging lengths k, m, n, and h.

$$k = \sqrt{x^2 + y^2 + z^2} \tag{1}$$

$$m = \sqrt{(d-x)^2 + (s-y)^2 + z^2} \tag{2}$$

$$n = \sqrt{x^2 + (s-y)^2 + z^2} \tag{3}$$

$$h = \sqrt{(d-x)^2 + y^2 + z^2} \tag{4}$$

In the previously publications by Filipovic at al. [5 and 6] and others, the motion trajectories of angles θ_1, θ_2 and θ_3 were presumed to start from zero. In this paper, because of the calibration of the system and realistic motion of these angles, the following has been defined:

a) the starting lengths of the cable wound on all three winches,
b) the real rotation direction of each motor based on the construction of the system, which is now different in comparison with the motor's rotation direction from the previously published papers.

This represents a real set of initial conditions for the motion of the system.

The novel geometry of the CPRRM system was defined by introduction of the winches for performing smooth cable winding/unwinding. Radii of all three winches, R_i, lengths of cables which connect the highest hanging points with winches lw_i, and all the other dynamic variables change their values during camera carrier's 3D motion.

In this Section, the changes of these variables will be included into the system's mathematical model. By introducing this novel phenomenon, a new form of the mathematical model of the CPRRM system is obtained. Based on Figure 2, one can define the length of each cable within the workspace of the camera. Length ρ_1 presents the complete length of the first cable within the workspace of the camera and it is defined by equation (5). Lengths ρ_2 and ρ_3 present complete lengths of the second and third cables within the workspace of the camera and these variables are defined by equations (6) and (7), respectively.

$$\rho_1 = k + n \tag{5}$$

$$\rho_2 = h \tag{6}$$

$$\rho_3 = m \tag{7}$$

For small changes of lengths defined by (5) - (7), one can write

$$\Delta\rho_1 = \Delta k + \Delta n \tag{8}$$

$$\Delta\rho_2 = \Delta h \tag{9}$$

$$\Delta\rho_3 = \Delta m \tag{10}$$

Resizing $\Delta\rho_1$ and Δlw_1, are caused by resizing $\Delta(\theta_1 * R_1)$. The same can be said for the other subsystems as follows.

$$\Delta(\theta_1 * R_1) = -\Delta\rho_1 - \Delta lw_1 \tag{11}$$

$$\Delta(\theta_2 * R_2) = -\Delta\rho_2 - \Delta lw_2 \tag{12}$$

$$\Delta(\theta_3 * R_3) = -\Delta\rho_3 - \Delta lw_3 \tag{13}$$

Equations (11) - (13) are crucial for defining the kinematic and also for defining the dynamic model of the observed CPRRM system. Substituting equations (8), (9), (10) into equations (11), (12), (13), respectively, we obtain.

$$\Delta(\theta_1 * R_1) = -(\Delta k + \Delta n) - \Delta lw_1 \tag{14}$$

$$\Delta(\theta_2 * R_2) = -\Delta h - \Delta lw_2 \tag{15}$$

$$\Delta(\theta_3 * R_3) = -\Delta m \ \ - \Delta lw_3 \tag{16}$$

If all three equations (14) - (16) are divided with a small increment of time Δt, in the limit one can write the next set of equations.

$$(\theta_1 \dot{*} R_1) = -(\dot{k} + \dot{n}) - \dot{lw_1} \tag{17}$$

$$(\theta_2 \dot{*} R_2) = -\dot{h} - \dot{lw_2} \tag{18}$$

$$(\theta_3 \dot{*} R_3) = -\dot{m} \ \ - \dot{lw_3} \tag{19}$$

Derivative of two variables of magnitude θ_i and R_i, is defined by

$$(\theta_\iota \dot{*} R_\iota) = \dot{\theta}_i * R_i + \theta_i * \dot{R}_i \tag{20}$$

By applying equation (20) in (17), (18) and (19) we get following

$$\dot{\theta}_1 * R_1 + \theta_1 * \dot{R}_1 = -(\dot{k} + \dot{n}) - \dot{lw_1} \tag{21}$$

$$\dot{\theta}_2 * R_2 + \theta_2 * \dot{R}_2 = -\dot{h} - \dot{lw_2} \tag{22}$$

$$\dot{\theta}_3 * R_3 \dot{+} \theta_3 * \dot{R}_3 = -\dot{m} \ \ - \dot{lw_3} \tag{23}$$

From equations (21) - (23) one can write the following equations for angular velocity: $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$, of all three winches.

$$\dot{\theta}_1 = -\frac{\dot{k}+\dot{n}}{R_1} - \frac{\dot{lw_1}}{R_1} - \frac{\theta_1 * \dot{R}_1}{R_1} \tag{24}$$

$$\dot{\theta}_2 = -\frac{\dot{h}}{R_2} - \frac{\dot{lw_2}}{R_2} - \frac{\theta_2 * \dot{R}_2}{R_2} \tag{25}$$

$$\dot{\theta}_3 = -\frac{\dot{m}}{R_3} - \frac{\dot{lw_3}}{R_3} - \frac{\theta_3 * \dot{R}_3}{R_3} \tag{26}$$

From equations (1) - (4), one can derive the first derivatives of lengths *k, h, m*, and *n* and obtain variables: $\dot{k}, \dot{h}, \dot{m}$ and $\dot{n}$. They depend on camera carrier motion coordinates x, $\dot{x}$, y, $\dot{y}$, z, $\dot{z}$, and CPRRM system's workspace dimensions d, s. If these variables $\dot{k}, \dot{h}, \dot{m}$ and $\dot{n}$ are substituted in equations (24) - (26), it follows.

$$\dot{\theta}_1 = -\left(\left(\frac{x}{R_1 * k} + \frac{x}{R_1 * n}\right) * \dot{x} + \left(\frac{y}{R_1 * k} - \frac{(s-y)}{R_1 * n}\right) * \dot{y} + \left(\frac{z}{R_1 * k} + \frac{z}{R_1 * n}\right) * \dot{z}\right) - \frac{\dot{lw_1}}{R_1} - \frac{\theta_1 * \dot{R}_1}{R_1} \tag{27}$$

$$\dot{\theta}_2 = -\left(-\frac{d-x}{R_2 * h} * \dot{x} + \frac{y}{R_2 * h} * \dot{y} + \frac{z}{R_2 * \cdot h} * \dot{z}\right) - \frac{\dot{lw_2}}{R_2} - \frac{\theta_2 * \dot{R}_2}{R_2} \tag{28}$$

$$\dot{\theta}_3 = -\left(-\frac{d-x}{R_3 * m} * \dot{x} - \frac{s-y}{R_3 * m} * \dot{y} + \frac{z}{R_3 * m} * \dot{z}\right) - \frac{\dot{lw_3}}{R_3} - \frac{\theta_3 * \dot{R}_3}{R_3} \tag{29}$$

By comparing equations (27) - (29) with equations (7) - (9) defined by Filipovic at al. [5], one can see that the kinematic model of CPRRM is far more complex. In equations (7) - (9) defined by Filipovic at al. [5], where variables $\dot{\theta}_i$ depend only on camera carrier motion coordinates x, $\dot{x}$,

y, $\dot{y}$, z, $\dot{z}$, RSCPR system's workspace dimensions d, s and radius of winch $R_i = const.$
The system of equations (27) - (29) can be written in matrix form

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix} = -\begin{bmatrix}\frac{x}{R_1*k}+\frac{x}{R_1*n} & \frac{y}{R_1*k}-\frac{s-y}{R_1*n} & \frac{z}{R_1*k}+\frac{z}{R_1*n}\\ -\frac{d-x}{R_2*h} & \frac{y}{R_2*h} & \frac{z}{R_2*h}\\ -\frac{d-x}{R_3*m} & -\frac{s-y}{R_3*m} & \frac{z}{R_3*m}\end{bmatrix} * \begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} - \begin{bmatrix}\frac{l\dot{w}_1+\theta_1*\dot{R}_1}{R_1}\\ \frac{l\dot{w}_2+\theta_2*\dot{R}_2}{R_2}\\ \frac{l\dot{w}_3+\theta_3*\dot{R}_3}{R_3}\end{bmatrix} \tag{30}$$

From equations (30), one can see that angular velocities of considered winch $\dot{\theta}_i$ are dependent on variables: camera carrier motion coordinates x, $\dot{x}$, y, $\dot{y}$, z, $\dot{z}$, workspace dimensions d, s, of considered RSCPR system, variable R_i as well as variables $\dot{R}_i, l\dot{w}_i, \theta_i$.
Let vector E and matrix J_M be defined respectively by

$$E = \begin{bmatrix}\frac{l\dot{w}_1+\theta_1*\dot{R}_1}{R_1}\\ \frac{l\dot{w}_2+\theta_2*\dot{R}_2}{R_2}\\ \frac{l\dot{w}_3+\theta_3*\dot{R}_3}{R_3}\end{bmatrix} \tag{31}$$

and

$$J_M = \begin{bmatrix}\frac{x}{R_1*k}+\frac{x}{R_1*n} & \frac{y}{R_1*k}-\frac{s-y}{R_1*n} & \frac{z}{R_1*k}+\frac{z}{R_1*n}\\ -\frac{d-x}{R_2*h} & \frac{y}{R_2*h} & \frac{z}{R_2*h}\\ -\frac{d-x}{R_3*m} & -\frac{s-y}{R_3*m} & \frac{z}{R_3*m}\end{bmatrix} \tag{32}$$

Then, equation (30) can be written in a more compact form

$$\dot{\phi} = -J_M * \dot{p} - E \tag{33}$$

By comparing equations (30) from this paper with the respective equations (10) from paper written by Filipovic at al. [5], one can see that the system from this paper is much more complex than the system described by Filipovic at al. [5]. Jacobi matrix J_S in equation (10) from [5] is ostensibly similar with the matrix J_M from this paper. It needs to be emphasized that, in equations (30) from this paper, matrix J_M depends on variables R_1, R_2 and R_3 which now change their values during the CPRRM system's task implementation.

In the case of RSCPR system defined by Filipovic at al. [5], the magnitudes R_1, R_2 and R_3, in matrix J_S, are constant during camera carrier's motion.

Vector E describes the influence of the cable winding/unwinding process on the winch. It is dependent on the following variables: $\dot{R}_i, l\dot{w}_i, \theta_i$ and R_i. More precisely, one can say that the kinematic model of the CPRRM system from Figure 2 is dependent on: variables $x, y, z, \dot{x}, \dot{y}, \dot{z}$, dimensions of the system's workspace d, s, variable R_i, and variable $\dot{R}_i, l\dot{w}_i, \theta_i$.

Compare the equations (30) from this paper with the respective equations (5) from paper written by Kevac and Filipovic [9]. The kinematic model of cable winding/unwinding system, which includes only one actuator system (motor, gear and winch) defined by Kevac and Filipovic [9] by equation (5). By comparing equation (5) from paper defined by Kevac and Filipovic [9], with the equation (30) in this paper, we can see how

1. workspace dimensions d and s, of the CPRRM system,
2. selected construction of considered CPRRM system is characterized by strong coupling

between external coordinates p, $\dot{p}$ and internal coordinates ϕ, $\dot{\phi}$,

3. selected novel construction of the winch with dynamics variables: R_i, $\dot{R}_i, lw_i, \dot{lw}_i$, affect the complexity of the kinematic model of the CPRRM system.

It is clear that the complication of the kinematic model will strongly affect the complexity of the dynamic model of the CPRRM system. The dynamic model of the CPRRM system is subject of our research in next subsection.

2.2. The dynamic model of the CPRRM system

In the previous Subsection, the equations which describe the kinematic model of the CPRRM system have been defined. The kinematic model of this system presents a prerequisite for performing dynamic analysis of the system. In order to define the dynamic model of the CPRRM system one needs to identify the resultant torques acts on shaft of each winch of cable winding/unwinding subsystems. Lagrange virtual work principle is used in this paper to acquire the following equation:

$$F^T * \dot{p} = {M_M}^T * \dot{\phi} \tag{34}$$

where $F = \left[F_x \, F_y \, F_z\right]^T$- vector of forces acting on the camera carrier, and $M_M = [M_{M1} \, M_{M2} \, M_{M3}]^T$ - vector of the resultant torques acting on shafts of all three cable winding/unwinding subsystems. By substituting equation (33) into equation (34), the following equation is obtained

$$F^T * \dot{p} = -{M_M}^T * (J_M * \dot{p} + E) \tag{35}$$

In order to establish a relation between M_M and F in equation (34), a different formulation of equation (34), without changing the essence of this equation, will have to be made. For this purpose, the following diagonal matrices are defined

$$P_d = diag(\dot{p}) \tag{36}$$

$$E_d = diag(E) \tag{37}$$

This mathematical formulation does not change the essence of the results. In this way one obtains a reorganized form of equation (35), i.e.

$$F^T * P_d = -{M_M}^T * (J_M * P_d + E_d) \tag{38}$$

Equations (35) and (38) are identical. Now we can formulate a vector of forces F acting on the camera carrier, by dividing the equation (38) by matrix P_d

$$F^T = -{M_M}^T * (J_M + E_d * (P_d)^{-1}) \tag{39}$$

Then, if the equation (39) is transposed

$$F = -(J_M + E_d * (P_d)^{-1})^T * M_M \tag{40}$$

and finally, the equation for calculation of the resultant torques M_M

$$M_M = -((J_M + E_d * (P_d)^{-1})^T)^{-1} * F \tag{41}$$

The final form of the mathematical model of the CPRRM system, which is analyzed in this paper, can be written. However, in this paper the vector of resultant torques M_M acting on shafts of all three-cable winding/unwinding subsystems includes the dynamics of the process of cable smooth winding/unwinding on the new construction of the winch. It can be seen that the vector of resultant torques M_M is dependent on geometry and kinematic characteristics of the considered CPRRM system, equations (34) - (41), i.e. it depends on: variables $x, y, z, \dot{x}, \dot{y}, \dot{z}$, dimensions of the system's workspace d, s, variable R_i, $\dot{R}_i, \dot{lw}_i, \theta_i$ and vector of forces acting on the camera carrier F .

The dynamic model of the system is presented by vector equation (42)

$$u = G_v * \ddot{\phi} + L_v * \dot{\phi} + S_v * M_M \tag{42}$$

where, $u = [u_1\ u_2\ u_3]^T$ - control signals (voltages) of all three motors, $G_{v(3x3)} = diag(G_{v1}\ G_{v2}\ G_{v3})$ - matrix that carries the information about motors' inertia, $G_{vi} = (J_{ri} * R_{ri})/C_{Mi}$, $L_{v(3x3)} = diag(L_{v1}\ L_{v2}\ L_{v3})$ - matrix that carries the information about motors' damping, $L_{vi} = \frac{R_{ri} * B_{Ci}}{C_{Mi}} + C_{Ei}$, $S_{v(3x3)} = diag(S_{v1}\ S_{v2}\ S_{v3})$ - matrix that carries the information about motors' geometric characteristics, $S_{vi} = R_{ri}/C_{Mi}$, while $\dot{\phi}$ and $\ddot{\phi}$ present the first and the second derivatives of angular positions of all three winches, respectively. Regardless of the complexity of the CPRRM system, it has three DOF (degrees of freedom) and its model is defined by three scalar equations which are presented in vector form by equation (42).

The mathematical model of the CPRRM system is defined by a vector equation (42) where, in addition to the variables: $x, y, z, \dot{x}, \dot{y}, \dot{z}, d, s, R_i, \dot{R}_i, \dot{lw}_i, \theta_i, F_x, F_y, F_z$ and the parameters of the motors G_v, L_v, S_v, the first and the second derivatives of the shaft angular positions of all three winches $\dot{\phi} = [\dot{\theta}_1\ \dot{\theta}_2\ \dot{\theta}_3]^T$ and $\ddot{\phi} = [\ddot{\theta}_1\ \ddot{\theta}_2\ \ddot{\theta}_3]^T$, respectively, and control signals of all three motors u_1, u_2, u_3, participate as well.

The equations (1) - (42) define the mathematical model of the CPRRM system. By using the new constructive solution of the winch, a smooth winding/unwinding process of the cable on the winch for all three actuators subsystems was ensured resulting in smooth motion of the camera. The mathematical model refers to the chosen construction of the CPRRM system. The construction of the CPR system itself has its own geometry that affects the kinematic and the dynamic model. Each new CPR system construction needs to be thoroughly analyzed and modeled by recognizing all the specific characteristics.

In this paper, we can see how number of selected parameters of CPR system such as

1. workspace dimensions d, s of the CPRRM system,
2. construction of considered CPRRM system, is characterized by strong coupling between external coordinate p, $\dot{p}$ and internal coordinate ϕ, $\dot{\phi}$,
3. new construction of the winch with dynamics variable $R_i, \dot{R}_i, lw_i, \dot{lw}_i$,
4. actuator characteristics,
5. gear characteristics,
6. camera carrier velocity,
7. camera carrier capacity,
8. reference trajectory,
9. controller,
10. strong coupling between all three actuator subsystems,

affect the complexity of the dynamic response of the CPRRM system.

It is clear that the complexity of the kinematic model has strongly affected the complexity of the dynamic model of the CPRRM system, but we also have a number of dynamic features that strongly influence the response of the CPRRM system, as well.

In order to prove validity of the mathematical formulations defined in this Section, in the following Section a novel program package, named CPRRMSOFT, is presented.

3. The Program Package CPRRMSOFT

Based on the mathematical model of the CPRRM system, defined by equations (1) - (42), a novel program package CPRRMSOFT has been synthesized. This program package contains several subroutines combined into one unit.

1. Subroutine for generation of the reference trajectory. In this case, the reference trajectory of the camera carrier in Cartesian space (*x, y, and z*) is defined. From the Cartesian space the internal coordinates, angular positions of three winding/unwinding subsystems ($\theta_1, \theta_2, \theta_3$) (as defined by equation (30)), are calculated. This procedure includes the kinematic model of the CPRRM system which consists of three cable smooth winding/unwinding subsystems - these subsystems include the new constructive solution of the winch which is shown in Figure 1.
2. Subroutine for generation of the dynamic response of the CPRRM system. In this subroutine, the influence of the changes of radii R_i and lengths lw_i between the winches and hanging points is included. This is defined through the resultant torque M_M which includes the dynamics of camera carrier's motion (force *F*) via Lagrange virtual work principle. Equation (41) includes the geometry of the mechanism, i.e. its kinematic model, as well. It can be seen that now the relation between the resultant torques M_M and forces F are related via the following: variables $x, y, z, \dot{x}, \dot{y}, \dot{z}, \dot{R}_i, \dot{lw}_i, \theta_i$, and R_i and dimensions of system's workspace d and s. Characteristics of the selected motors and gearboxes have a significant influence on the dynamic response of the CPRRM system.
3. Subroutine for control structure of the system. This routine assumes the creation of various control algorithms for the motion control of the camera carrier in 3D space.

Based on the analysis from Sections 2 and 3, the next part of the paper will show the simulation experiments which illustrate implementation of one 3D trajectory of the novel CPRRM system.

4. Testing the CPRRM System - the Simulation Experiments

In this Section one example of a complex motion of the new CPRRM system's camera carrier is simulated. As can be seen from Figure 2, the CPRRM system consists of three cable winding/unwinding subsystems. The winch shown in Figure 1 is used. The mathematical model of the CPRRM system described in Section 2 will be tested. For generation of all the results in this Section, the program package CPRRMSOFT described in Section 3 is used.
The CPRRM system has the following workspace dimensions: $d \cdot s \cdot v = 0.50 \cdot 0.44 \cdot 0.50\ (m)$ and carrying capacity of 0.319 kg.

For the purpose of testing, the following reference trajectory has been used: the camera carrier needs to move along a straight line from point $Cpstart = [0.1\ 0.1 - 0.3]^T$ to point $Cpend = [0.4\ 0.31 - 0.1]^T$. Velocity of the camera carrier has a trapezoidal shape with the maximal value of $0.83\ (\mathrm{m}/s)$. For driving all the winches, Faulhaber's motor of type 2642W012CR, see reference [10], is used. This motor has rated voltage of 12 (V). For the purpose of control of the CPRRM system, a classic PID controller with a feedback in terms of angular positions of all three winches has been used. The same PID controller for all three actuators has been used; the PIDs have the following gains: $K_P = 75.5$, $K_I = 10.5$ and $K_D = 0.9$.

Figure 3 shows the first result - composite velocity of the camera carrier. The reference velocity along with the real velocity is presented as functions of time. From this figure, one can see that the velocity has a trapezoidal shape and that the reference and real velocities mainly overlap which shows a good tracking of the camera's reference velocity. Figure 4 presents the control signals of all three motors. It is important that these three signals do not exceed the rated value, i.e. ± 12 (V). From Figure 4 one can see that these signals are far below the limit values, which indicates that the motors are chosen well and that there are significant margins of stability.

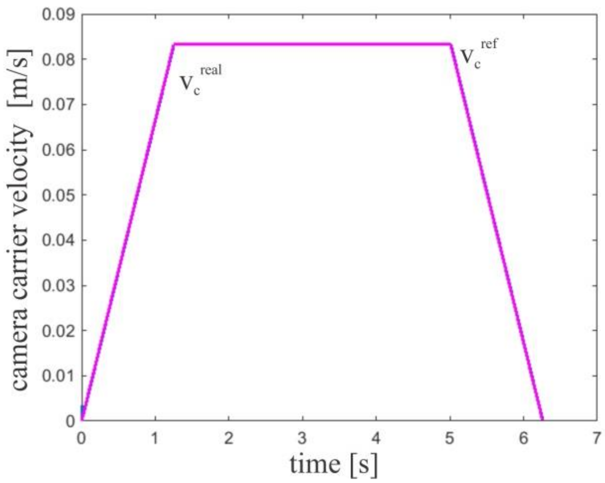

Figure 3. The real and reference composite camera carrier's velocity v_c.

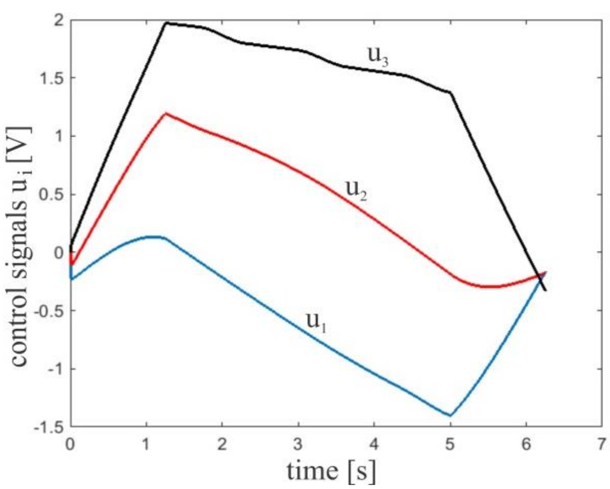

Figure 4. Control signals.

The next result is shown in Figure 5 - the angular positions of all three subsystems motor - gear - winch, i.e. angles θ_1, θ_2, and θ_3. The real and reference values are shown in this figure and one can see that these trajectories are smooth which is important for a regular behavior of the system. Figure 6 shows differences between the real and reference angular positions.

From this figure one can see that the real system closely follows the reference value. Figure 7 presents angular velocities of all three winch shafts: $\dot{\theta}_1$, $\dot{\theta}_2$ and $\dot{\theta}_3$. This figure presents the real and reference values and one can see that the system follows well the reference trajectory.

Figure 8 shows the reference and real positions of the camera carrier during implementation of the task in the space of Cartesian coordinates of the camera carrier: *x, y*, and *z.* Figure 9 shows differences between the real and reference values of camera carrier's position and one can see that a good tracking has been achieved.

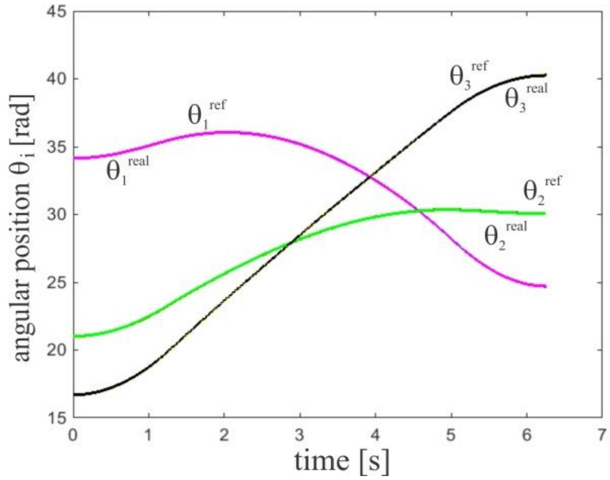

Figure 5. Angles: θ_1, θ_2 and θ_3.

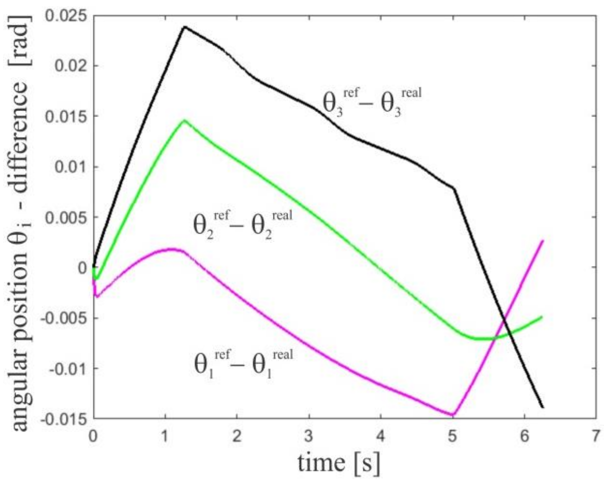

Figure 6. Differences between the real and reference values of angles θ_1, θ_2 and θ_3.

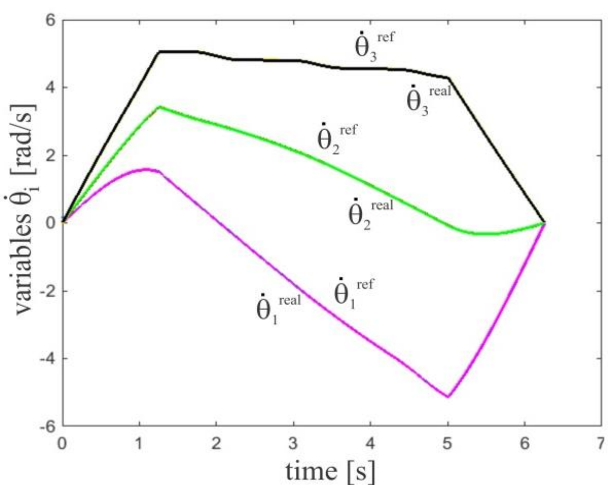

Figure 7. The first derivatives of angles: θ_1, θ_2 and θ_3.

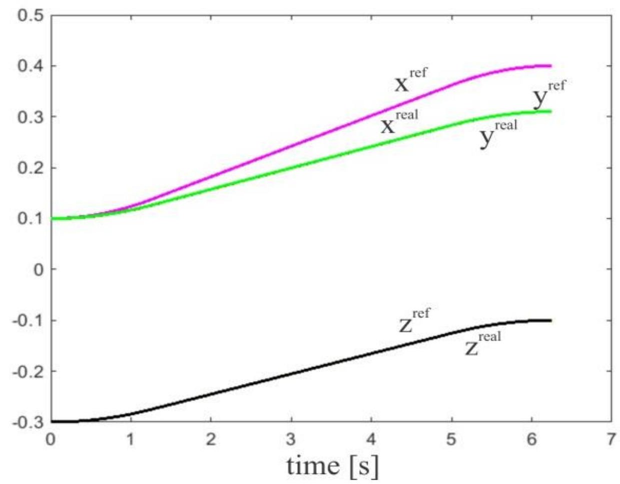

Figure 8. Camera carrier's position *x, y, z.*

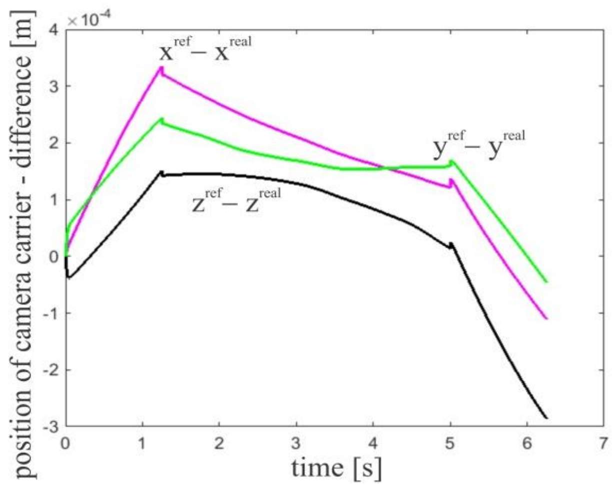

Figure 9. Deviation real to the reference position *x, y* and *z.*

The next results, shown in Figure 10, are the first derivatives of variables *x, y* and *z.* In this figure one can also see that a good tracking of reference trajectory has been achieved.

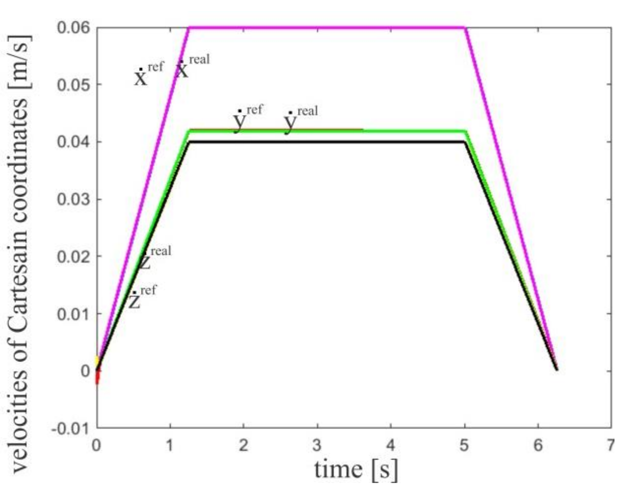

Figure 10. Variables $\dot{x}, \dot{y}$ and $\dot{z}$.

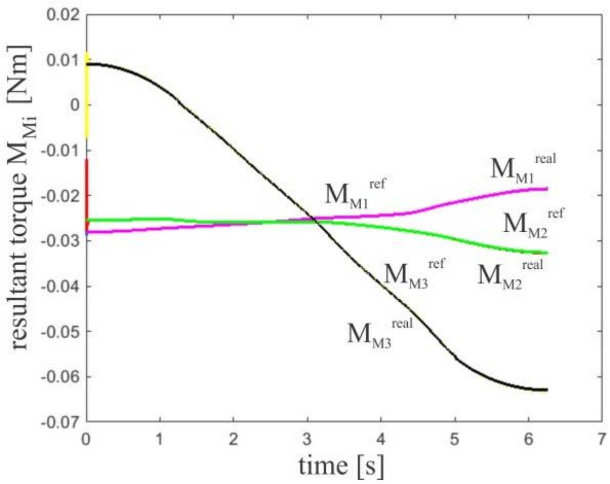

Figure 11. Resultant torques M_{M1}, M_{M2} and M_{M3}.

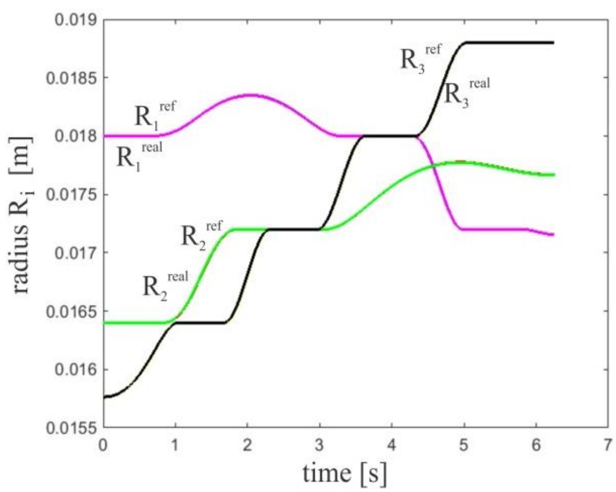

Figure 12. Radii R_1, R_2 and R_3.

Further on, Figure 11 presents the real and reference resultant load torques of all three subsystems motor - gear - winch. From this figure one can see a good tracking of the reference values; it can be seen that real torques have mild oscillations only at the initial moments of execution of the task. This can be resolved by introducing different control structures or by selecting different combinations of PID controller's gains.

The next figure, Figure 12, presents the change of winding/unwinding radii of all three winches, i.e. variables R_1, R_2, and R_3, while Figure 13 shows the first derivatives of these variables - $\dot{R}_1$, $\dot{R}_2$, and $\dot{R}_3$.

At the beginning, from 0 - 0.8(s), the first winch is winding inside the *con* area, where its radius R_1 has a constant value of R_1= 0.018(m), see Figures 12 and 13, while its first derivative is $\dot{R}_1$= 0. After this moment, from 0.8 - 2(s), the first winch is inside the *smvar* area and during this period of time radius R_1 grows and its first derivative is $\dot{R}_1$> 0. After that moment, from 2 - 3.2 (s), the first cable starts to unwind and in that period of time, radius R_1 decreases towards its first value of 0.018(m), while $\dot{R}_1$< 0. Then, during the period from 3.2 - 4.2 (s), the first winch returns to the same *con* area and then radius R_1 has a constant value - 0.018(m) and its first derivative is zero. In the period 4.2 - 5.2 (s), the first winch enters the *smvar* area where the first cable is unwinding, radius R_1 decreases from the value of 0.018(m) to 0.0172 (m) and its first derivative has a negative value. From 5.2(s) to 5.8 (s), the first winch enters the *con* area, which is characterized by the constant value of radius 0.0172 (m). That is the period when the system starts to brake and from the 5.8 (s) it enters the *smvar* area, where radius R_1 slightly decreases until the end of the motion. In that period of time one can see that $\dot{R}_1 < 0$.

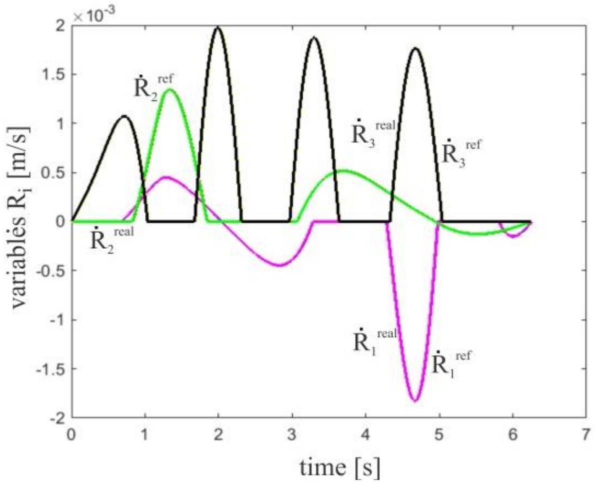

Figure 13. The first derivatives of radii R_1, R_2 and R_3.

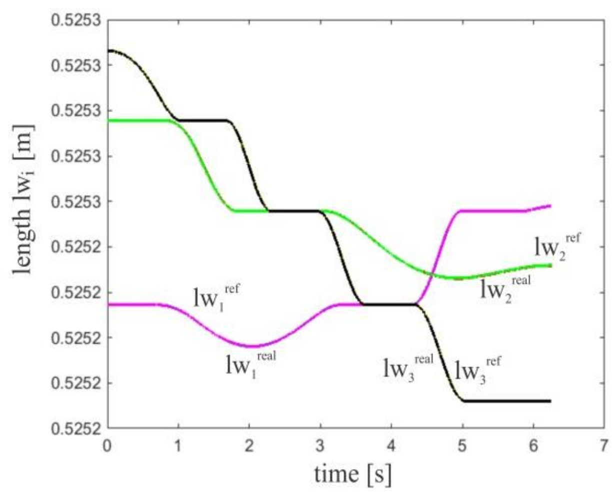

Figure 14. Lengths: lw_1, lw_2 and lw_3.

Radius R_2 has different dynamics in comparison with radius R_1, see Figures 12 and 13. This is caused by the fact that the second winch enters areas *con* and *smvar* in different time periods compared to R_1. Radius R_3 has different dynamics in comparison with both radii, R_1 and R_2. For a desired motion of the camera carrier in 3D space, it is essential that the actuators motor-gear-winch have coordinated changes of their characteristics. This requirement is satisfied, as can be seen through the changes of winding/unwinding radii R_1, R_2, and R_3. By comparing Figures 12 and 5, the analogy between the changes of radii R_1, R_2, and R_3 and changes of angles θ_1, θ_2, and θ_3 can be noticed.

As radii R_1, R_2, and R_3 change their values during the implementation of angles θ_1, θ_2, and θ_3, the lengths lw_1, lw_2, and lw_3 and angles γ_1, γ_2, and γ_3 change their values as well.

Figure 14 shows the changes of variables lw_1, lw_2, and lw_3, while Figure 15 shows the first derivatives of these variables $\dot{lw}_1$, $\dot{lw}_2$, and $\dot{lw}_3$. From these figures one can see that these variables have smooth changes in time. By comparing the Figures 14 and 12, one can see that during the *con* period both R_i and lw_i are constant. Unlike *con* period, when *i*-th winch is in *smvar* period during the cable winding, radius R_i is growing while length lw_i is decreasing in the same period. Analogously, when i-th winch is in *smvar* area during the cable unwinding process, radius R_i is decreasing, while length lw_i is growing in this period. This conclusion is confirmed by comparing their first derivatives from Figures 15 and 13.

The last figure of this section, Figure 16 shows the changes of angles γ_1, γ_2, and γ_3 and one can see that these variables have the same dynamics of change as those of lengths lw_1, lw_2, and lw_3, i.e. when cable is winding both lw_i and γ_i decrease their values and vice versa. When i-th winch is in *con* area, i.e. when $R_i = const$, angle γ_i also has a constant value, as well as length lw_i.

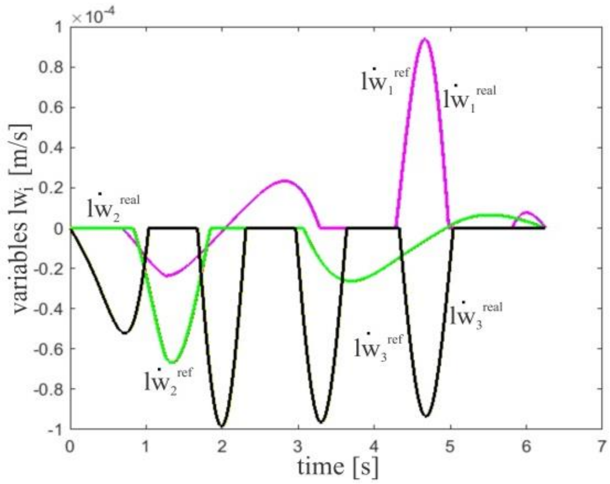

Figure 15. The first derivatives of lengths $\dot{lw}_1$, $\dot{lw}_2$ and $\dot{lw}_3$.

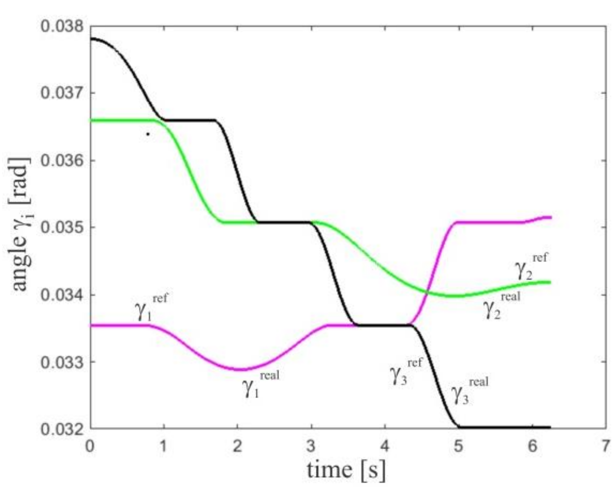

Figure 16. Angles: γ_1, γ_2 and γ_3.

In all the figures from this Section, one can see clearly that the real system follows the reference trajectory and that all the variables are smooth. For this reason, motion controlling of the camera carrier of the complex CPRRM system is relatively straightforward to accomplish.

5. Conclusions

By thorough analysis of the CPR system, it was shown those characteristics of winch: radius and length of cable winding/unwinding, are very influential on the system's dynamics response. It was shown that if these characteristics have "jumpy" change dynamics; it can highly affect oscillatory behavior and instability of the CPR system. Because of that, new construction of the Cable suspended Parallel Robot using novel construction of the winches for performing single - row Radial Multi-layered smooth cable winding/unwinding, named CPRRM system, was formed. This system's characteristic variables: cable winding/unwinding radius and length and their first derivatives are smooth and nonlinear. This turns out to be very important, usable, component in dynamics response of the whole CPRRM system.

Very important contribution of this paper is setting geometric relationships between changeable cable lengths of the CPRRM system, which is given by equations (14)-(16). These equations are affected by novel winch dynamics variables as well

- radius R_i, and
- cable winding/unwinding length lw_i.

Importance of these equations lays in the fact that new forms of kinematic and dynamic models of observed CPRRM system are derived from them: equations (30) and (42), respectively.

For validation and verification of the concepts implemented in formulation of the mathematical model of CPRRM system, a novel program package CPRRMSOFT is generated. By using this program package, the simulation experiments for a chosen trajectory of the CPRRM system's camera carrier in camera's 3D workspace were performed for the purpose of verification of the presented mathematical principles. By choosing realistic parameters of the system and dynamic parameters of the applied motors, good simulation results have been obtained. It was clearly shown that a real system follows the reference trajectory. Their smooth change during the implementation of the task gives the assurance that camera carrier motion of the system will be smooth and easily controllable.

The results of this work are widely applicable because the new winch design can be incorporated into any cable-guided robotic system. This paper is not only theoretical, but it uncovers all the steps for researcher, designer, constructor and user to make a complex CPR system.

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