Heat and Mass Transfer Research Journal
Vol. 3, No. 2; 2019
Published by CanSRG
http://cansrg.com/journals/hmtrj/

MHD Non-Newtonian Casson Fluid Flow with Magnetic Field Effects Over A Non-Linear Stretching Sheet

Alfunsa Prathiba and Bandari Shankar*
Department of Mathematics, CVR College of Engineering, Ibrahimpatnam, Telangana, 501510, India

Abstract

In this paper, the boundary layer flow of a Magneto Hydrodynamics (MHD) non-Newtonian Casson fluid and the transfer of heat and mass were considered due to a non-linearly stretching sheet in the presence of heat source/sink and chemical reaction influences. Appropriate similarity transformations were employed to convert the governing partial differential equations to a set of highly non-linear ordinary differential equations and solved numerically by using fourth order Runga Kutta method along with shooting technique. In the current work, the impacts of the governing non-dimensional parameters on the velocity, temperature and concentration profiles were discussed and presented graphically. The profiles of temperature and concentration were found rising when the Casson parameter rose, but the opposite was true for velocity distribution. At the same time, it is observed that, the velocity profile got decreased at a fixed point with an increase of the non-linear stretching parameter, but the temperature profile was enhanced when the non-linear stretching parameter rose. In addition, to check the validity of the method, the coefficient of skin friction and temperature gradient was compared with the previously published work and found in a good agreement.

Keywords: *MHD; Heat and mass transfer; Non-Newtonian Casson fluid; Stretching sheet; heat source/sink; chemical reaction.*

1. Introduction

The behavior of non-Newtonian fluid flow over a stretching sheet on a boundary layer has been investigated by several researchers over the past few decades, because of its great importance in the industrial and technological applications. A few of its applications in the industrial process are like drilling muds, manufacturing coated sheets, extruding and unending polymer sheet from a die, spinning of fibers, etc. The rate of cooling or heating at the non-linear stretching sheet has an expected significance to produce the quality of the final product. The Casson fluid is a shear thinning fluid that belongs to a class of non-Newtonian fluid model. In a Casson fluid, since there is an infinite viscosity at zero rate, no flow occurs, but reveals zero viscosity at an infinite shear rate. Such type of fluid model can be conceived for the flow of tomato sauce, honey, different kind concentrated soups, jelly, etc. In certain cases, human blood flow is also addressed with a Casson fluid model, for the precision and accuracy in analysis. Indeed, a number of studies have been conducted on the boundary layer flow of non-Newtonian fluid, both theoretically and practically.

The first person who examined a problem on a stretching sheet by considering the flow of a fluid over a linearly stretched surface was Crane [1]. Bhattacharyya [2] examined a flow on a boundary layer and the transfer of heat over an exponentially shrinking sheet. Soon after, a Casson fluid flow problem with MHD over an exponentially shrinking sheet has been solved

* Corresponding Author
Submitted: April 3, 2019 Revised: October 25, 2019 Accepted: December 4, 2019

analytically using the Adomian Decomposition Method (ADM) by Nadeem et al. [3]. Pramanik [4] investigated a non-Newtonian fluid flow on a boundary layer conveyed by heat transfer due to an exponentially stretching surface in the presence of Suction or blowing at the surface. He found that an increase of the suction parameter enhances the coefficient of skin friction. Furthermore, Bala and Reddy [5] studied an MHD convective, two-dimensional steady Casson fluid flow on a boundary layer over an exponentially inclined permeable stretching surface. MHD flow of a Casson fluid model with inconsistent viscosity on a stretching surface with changeable thickness by employing Cattaneo-Christov heat flux model is employed instead of Fourier's law to discover the characteristics of heat transmit has been studied numerically using Keller box method by Malik et al. [6]. Alternatively, Bhattacharyya et al. [7] studied analytically an MHD boundary layer flow of a Casson fluid on a stretching/shrinking permeable sheet in the presence of wall mass transfer. Kameswaran et al. [8] investigated a non-Newtonian incompressible Casson fluid flow on a stagnation point over a stretching plate by considering Soret and Dufour effects.

A non-Newtonian Casson fluid flow in a boundary layer followed by the transmit of heat and mass towards a porous exponentially stretching sheet amid by velocity and thermal slip state in the attendance of thermal radiation, suction/blowing, viscous dissipation, heat source/sink and chemical reaction effects was investigated by Saidulu and Lakshimi [9]. They obtained that the velocity distribution decreases with an increase of Casson parameter, but the reverse process happens for concentration and temperature profiles. Shehzad et al. [10] examined mass transfer effects in the MHD flow of a Casson fluid over a porous stretching sheet in the incidence of a chemical reaction. They found that both Hartman number and Casson parameter have the same impacts on the velocity in a qualitative sense. Recently, Jakati et al. [11] solved an MHD non-Newtonian Cassonnanofluid flow over a stretching sheet using Homotopy Analysis Method (HAM).

All the above studies were concentrated on a linear stretching surface. Thus, the studies on a non-linearly stretching surface were explained as the following. Alinejad and Samarbakhsh [12] investigated numerically the flow and the properties of heat transport of incompressible viscous flow over a nonlinearly stretching sheet with the attendance of viscous dissipation. Sumalatha and Bandari [13] studied a Casson fluid a flow over a non-linearly stretching surface with an impact of heat source/sink and radiation. Adding to them, recently, a flow on a boundary layer and heat transport of a Casson fluid over a non-linearly permeable stretching surface with the effect of chemical reaction in the occurrence of a variable magnetic field was investigated by Reza et al. [14]. In their study, thermal radiation effects were considered so as to control the rate of heat transfer at the surface. Furthermore, the following researchers did their study on a non-linearly stretching surface [15]-[19].

As per the knowledge of the author, a magnetic field effect on a Casson fluid flow over a non-linearly stretching surface has not been studied widely. Thus, this study was conducted to fill the gap. Therefore, the purpose of this work was to analyze the impacts of a magnetic field and heat source/sink of Casson fluid flow over a nonlinearly stretching sheet. The governing, partial differential equations were transformed into nonlinear differential equations by using suitable similarity transformations and the equations were then solved by Runga Kutta fourth order method.

2. Mathematical formulation

In this work, an MHD incompressible viscous, steady, laminar, two-dimensional flow of a Casson fluid bounded by a non-linearly stretching sheet at $y = 0$ is considered. The flow is detained to $y > 0$ and it is directed in the $x-$ axis and normal to the $y - axis$. The equation of rheological state for an anisotropic and incompressible flow of the Casson fluid is given by

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + P_y/\sqrt{2\pi}\right)e_{ij}, \pi > \pi_{c,} \\ 2\left(\mu_B + P_y/\sqrt{2\pi_{c,}}\right)e_{ij}, \pi < \pi_{c,} \end{cases} \tag{1}$$

where μ_B and P_y are the plastic dynamic viscosity, yield stress, respectively. Correspondingly, π is the product of the constituent of deformation rate with itself, $\pi = e_{ij}e_{ij}$, e_{ij} is the (i,j)th component of the deformation rate and π_c is a critically value of this product based on the non-Newtonian model.

The equations of continuity, momentum, energy, and concentration governing such types of flow are given as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = V\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 \mathrm{u}}{\partial \mathrm{y}^2} - \frac{\sigma \mathrm{B}^2(\mathrm{x})}{\rho}\mathrm{u} \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q^*(x)}{\rho c_p}(T - T_\infty) \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_0(C - C_\infty) \tag{5}$$

where u and v are the velocity components in the x and y directions, respectively, V is the kinematic viscosity, ρ is the fluid density, $\beta = \mu_B\sqrt{2\pi_c}\,/P_y$ is the Casson fluid parameter, and k is the thermal conductivity of the fluid, D is the mass diffusivity coefficient and k_0 is the coefficient rate of chemical reaction.

The appropriate boundary conditions for the problem are given by

$$u = U = cx^n, = 0,\ T = T_\omega, C = C_\omega \text{ at } y = 0 \tag{6}$$

$$u \to 0\ ,\ T \to T_\infty, C \to C_\infty, \text{ as } y \to \infty \tag{7}$$

here, $c(c > 0)$ is a parameter related to the surface stretching speed, T_ω is the uniform temperature at the sheet, C_ω is the uniform concentration at the sheet, T_∞ and C_∞ are the free stream temperature and concentration, respectively, and n is the power index related to the surface stretching speed.

Introducing the following similarity transformations

$$\eta = y\sqrt{\frac{c(n+1)}{2V}}\ x^{\frac{n-1}{2}}, u = cx^n f',\ v = -\sqrt{cV(\frac{n+1}{2})}\ x^{\frac{n-1}{2}}\left[f + \frac{n-1}{n+1}\eta f'\right],\ \frac{T-T_\infty}{T_\omega - T_\infty} = \theta \text{ and } \frac{C-C_\infty}{C_\omega - C_\infty} = \phi \tag{8}$$

Substituting Eq. (8) into Eqs. (3)-(5), we obtain the reduced governing equations

$$\left(1 + \frac{1}{\beta}\right)f''' + ff'' - \frac{2n}{n+1}f'^2 - \frac{2}{n+1}Mf' = 0 \tag{9}$$

$$\theta'' + Pr\left(f\theta' + \frac{2}{n+1}\delta\theta\right) = 0 \tag{10}$$

$$\phi'' + Sc\left(f\phi' - \frac{2}{n+1}k_1\phi\right) = 0 \tag{11}$$

The boundary conditions take the following form

$$f' = 1, f = 0, \theta = 1, \phi = 1, \text{ at } \eta = 0 \tag{12}$$

$$f' \to 0, \theta \to 0, \phi \to 0, \text{ as } \eta \to \infty \tag{13}$$

where the prime indicates the differentiation with respect to η and $Pr = \frac{V}{k}$ is the Prandtl number, $\delta = \frac{Q_0 x}{U\rho c_p}$ is the heat source/sink parameter, $M = \frac{\sigma {B_0}^2}{\rho c x^{n-1}}$ is the magnetic field parameter, $Sc = \frac{V}{D}$ is the Schmidt number, and $k_1 = \frac{k_0 x}{U}$ is the chemical reaction parameter.

3. Numerical Analysis

The system of coupled nonlinear ordinary differential equations (9)-(11) along with the boundary conditions (13) and (13) are solved numerically by Runga Kutta fourth order method along with shooting technique. The proper value of ∞ (infinity) is choosen to be the most appropriate. Since the value of ∞(infinity) may change for another set of physical parameters. Once the value of ∞(infinity) is decided, the integration is carried out. Hence, for this problem we assumed ∞(infinity) to be 10. Applying this method on the given problem the following hypothesis are made

$$\left.\begin{array}{c} f \to f(1), f' \to f(2), f'' \to f(3), \;\; \theta \to f(4), \\ \theta' \to f(5), \phi \to f(6), \phi' \to f(7) \end{array}\right\} \tag{14}$$

and rewriting equations (8)-(10) with their boundary conditions in the form of

$$f''' = \left(-ff'' + \frac{2n}{n+1}f'^2 + \frac{2}{n+1}Mf'\right) / \left(1 + \frac{1}{\beta}\right) \tag{15}$$

$$\theta'' = -Pr\left(f\theta' + \frac{2}{n+1}\delta\theta\right) \tag{16}$$

$$\phi'' = -Sc\left(f\phi' - \frac{2}{n+1}k_1\phi\right) \tag{17}$$

The appropriate boundary conditions are

$$\left.\begin{array}{c} f_a(1) = 0, f_a(2 = 1), f_a(4) = 1, f_a(6) = 1, \\ f_b(2) = 0, f_b(4) = 0, f_b(6) = 0 \end{array}\right\} \tag{18}$$

The initial and boundary condition points are denoted by a and b i.e. $a = 0, b = \infty$. We assumed the step size $\Delta\eta = 0.01$ and the accuracy convergence criteria to be a five decimal value before solving the problem by the explained method using MATLAB program.

4. Results and Discussion

The effect of all the parameters involved in the governing equations for the non-linear stretching Casson fluid flow has been discussed in this work. The Runga Kutta fourth order method along with shooting technique is applied to solve the ordinary differential equations. The validity of this method is checked by comparing the coefficient of skin friction and temperature gradient for a viscous Newtonian fluid with the previously published papers as shown in Table-1 and Table-2, and the result is found in an excellent agreement.

Table 1. The evaluation of skin friction coefficient $f''(0)$ for $M = 0$.

n	$f''(0)$			
	Present result	Vajravelu [20]	Cortel [21]	Swati [15]
1	-1.000063	-1.0000	-1.0000	-1.0000
5	-1.194541	-1.1945	-	-1.1944
10	-1.234926	-1.2348	-1.234875	-1.2348

Table 2. Temperature gradient comparison $\boldsymbol{\theta'(0)}$ when $\boldsymbol{M = \delta = 0}$.

$\boldsymbol{n}$	$\boldsymbol{\theta'(0)}$		
	$\boldsymbol{Pr = 7}$		
1	Present Results	Vajravelu [20]	Swati [15]
5	-1.895402	-1.8953	-1-8953
10	-1.861581	-1.8610	-1.8611

The results obtained are explained through the following figures. Figures, 1(a), 1(b) and 1(c) exhibit the effects of the non-linear stretching parameter n for the velocity profile, temperature and concentration. As shown in the figure, it is found that an increase n enhances the velocity profile, but the profiles of temperature and concentration are reduced in the presence of a magnetic field parameter.

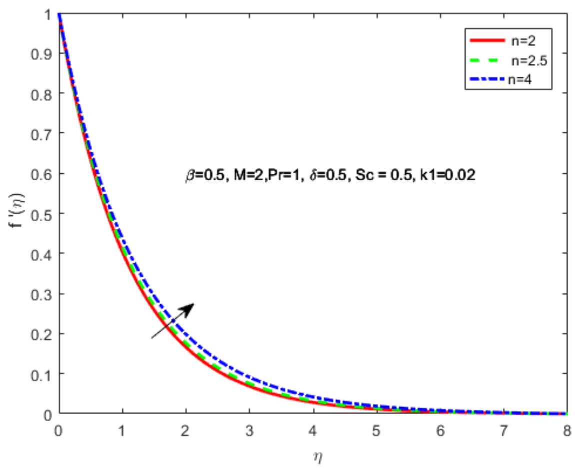

Figure 1(a). Velocity profile with variation of n.

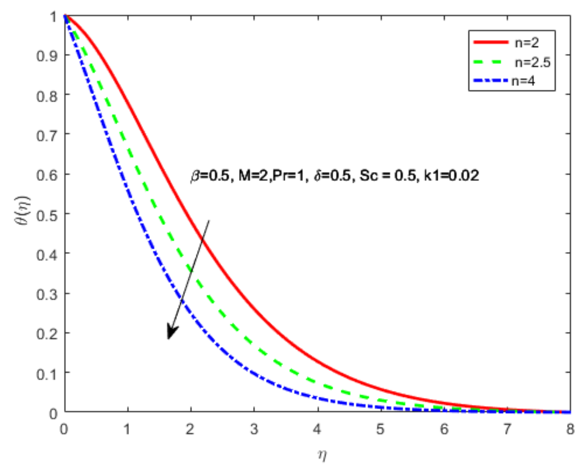

Figure 1(b). Temperature profile with variation of n.

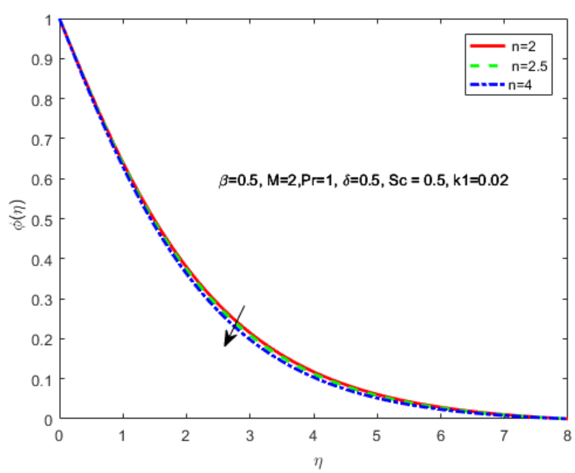

Figure 1(c). Concentration profile with variation of n.

Figures 2(a), 2(b) and 2(c) display the influence of the Casson fluid parameter β for the velocity, temperature, and concentration. The results obtained have shown that for a non-linear stretching parameter the temperature and concentration profiles increase, while the velocity field decreases with an increase of the Casson fluid parameter β. This is due to the fact that an increase of β results a rise in the viscosity of a fluid, i.e., the yield stress will be reduced. As a result, the boundary layer thickness of momentum will be reduced.

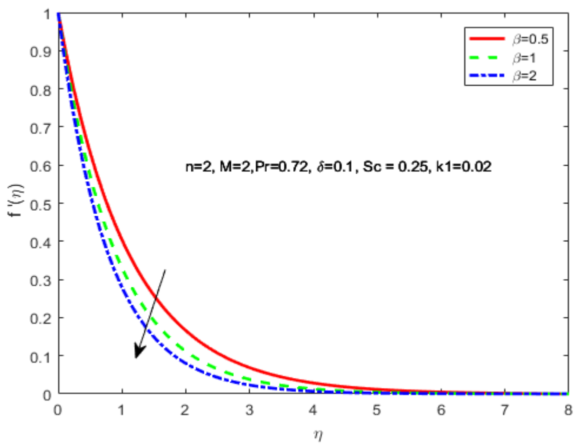

Figure 2(a). Velocity profile with variation of β.

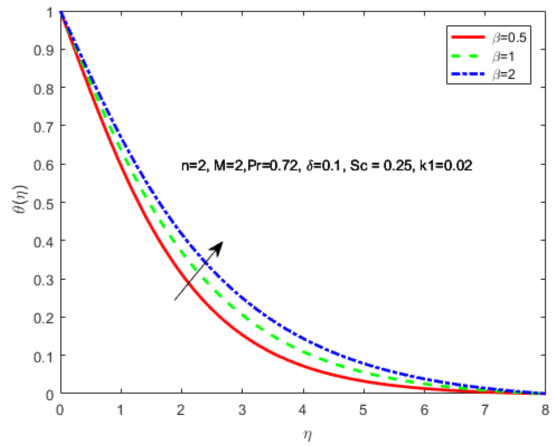

Figure 2(b). Temperature profile with variation of β.

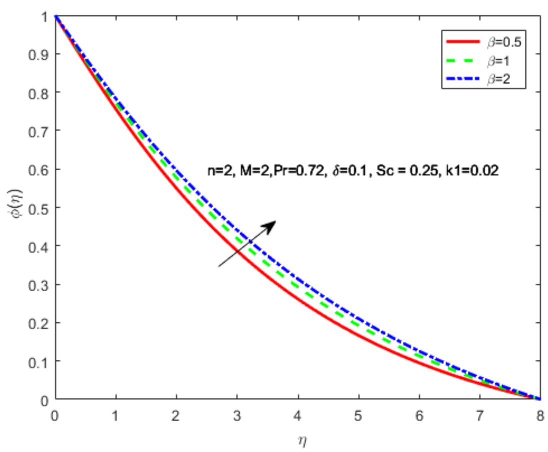

Figure 2(c). Concentration profile with variation of β.

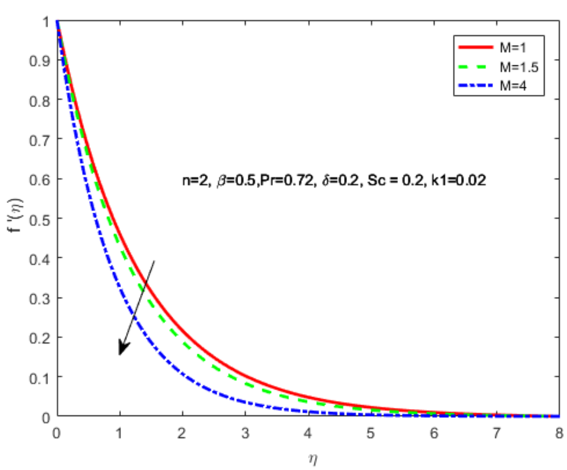

Figure 3(a). Velocity profile with variation of M.

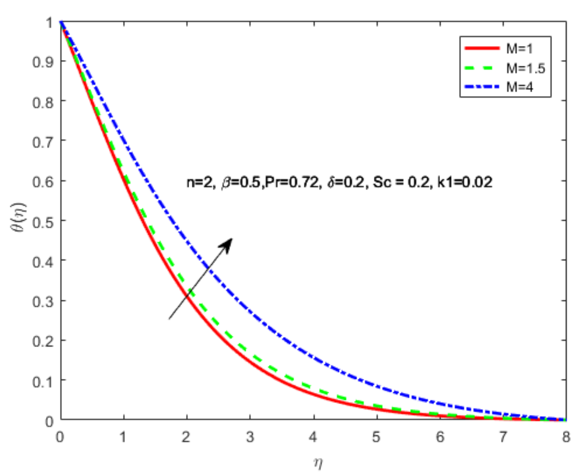

Figure 3(b). Temperature profile with variation of M.

Figures 3(a) and 3(b) demonstrate a magnetic field parameter effect on the velocity and temperature. The result described that the velocity profile decreases with an increase of M. Since the presence of M creates a Lorentz force that retards the motion of the fluid. This condition raises the thermal conductivity of the fluid and the momentum boundary layer thickness reduces.

The influence of Prandtl number Pr on the temperature profile is displayed using Figure 4. Thus, the distribution of temperature is reduced due to an increase of Pr. Since Pr has an inverse relationship with the thermal conductivity of the fluid and the thickness of thermal boundary layer also decreases.

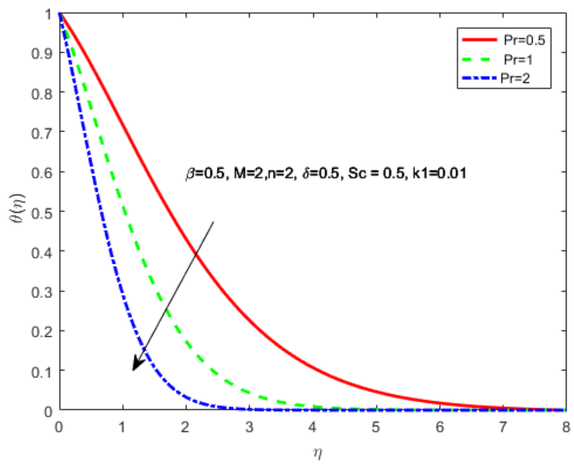

Figure 4. Temperature profile with variation of Pr.

The impact of heat source/sink parameter δ on the thermal boundary layer thickness is demonstrated through Figure 5. The result described that the thickness of the thermal boundary layer increases with an increase of δ. Physically, an increase in the heat source parameter can add more heat to the stretching sheet which increases its temperature.

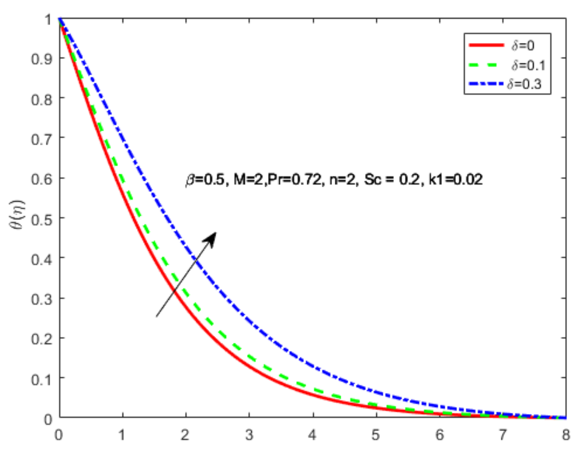

Figure 5. Temperature profile with variation of δ.

The influence of Schmidt number Sc and chemical reaction $k1$ for $n = 1$ on the concentration profile is displayed by using Figures. 6 and 7, respectively.

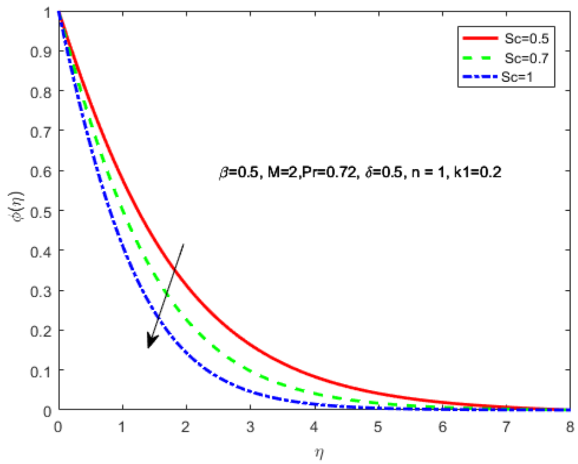

Figure 6. Concentration profile with variation of Sc.

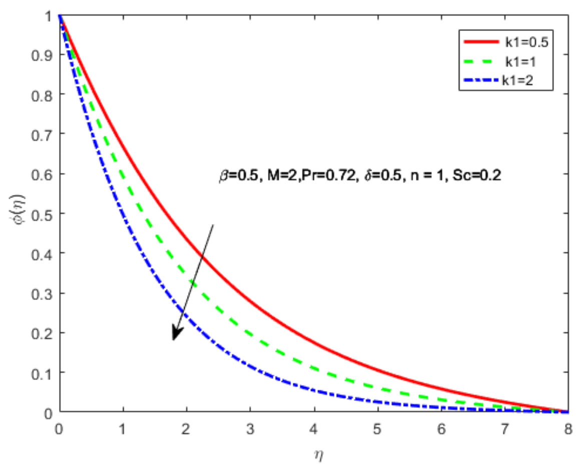

Figure 7. Concentration profile with variation of $k1$.

5. Conclusions

Considering the magnetic field in the Casson fluid and showing its impact on the flow nature, contributes a lot for further analysis. In addition, this work can be extended for the other problems involving porous media for the better understanding of the flow nature of Casson fluid. The numerical solution of Hydro magnetic two-dimensional boundary layer flow of a Casson fluid in the presence of heat source/sink and the chemical reaction influence, along with the characteristics, of heat and mass transfer over a nonlinearly stretching sheet were obtained.

Based on the numerical results the impact of various physical parameters is reviewed as,

1. An increase of the non-linear stretching parameter enhances the velocity at a fixed point, while the profiles of temperature and concentration reduce.
2. Velocity profiles reduce with an increase of magnetic field parameter.
3. The thermal boundary layer thickness decreases with an increase of heat source parameter and decreases with an increase of Prandtl number.
4. Concentration profile reduces with an increment of both Schmidt number and chemical reaction parameter.

References

[1] L. J. Crane, “Flow past a stretching plate, Zeitschrift fur Angew. Mathe-matik und Phys. ZAMP, vol 21(4), pp. 645-647, 1970.

[2] Bhattacharyya, Boundary Layer Flow and Heat Transfer over an Exponentially Shrinking Sheet, Chin. Phys. Lett., vol 28(7), pp. 074701(1-4), 2011.

[3] S. Nadeema, R. U. Haqa, and C. Lee, MHD flow of a Casson fluid over an exponentially shrinking shee, Sci. Iran., vol 19(6), pp. 1550-1553.

[4] S. Pramanik, Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation, Ain Shams Eng. J., vol. 5, pp. 205–212, 2014.

[5] P. Bala and A. Reddy, Magnetohydrodynamic flow of a Casson fluid over an exponentially inclined permeable stretching surface with thermal radiation and chemical reaction, Ain Shams Eng. J., vol. 7, pp. 593–602, 2016.

[6] S. Nadeema, R. U. Haqa, and C. Lee, MHD flow of a Casson fluid over an exponentially shrinking sheet, Sci. Iran., vol 19(6), pp. 1550-1553.

[7] S. Pramanik, Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation, Ain Shams Eng. J., vol 5, pp. 205-212, 2014.

[8] P. Bala and A. Reddy, Magnetohydrodynamic flow of a Casson fluid over an exponentially inclined permeable stretching surface with thermal radiation and chemical reaction, Ain Shams Eng. J., vol 7, pp. 593-602, 2016.

[9] M. Y. Malik, M. Khan, T. Salahuddin, and I. Khan, Variable viscosity and MHD flow in Casson fluid with Cattaneo-Christov heat flux model: Using Keller box method, Eng. Sci. Technol. an Int. J., vol 19, pp. 1985-1992, 2016.

[10] Krishnendu Bhattacharyyaa Tasawar Hayatb and A. Alsaedic, Analytic solution for magnetohydrodynamic boundary layer flow of Casson fluid over a stretching/shrinking sheet with wall mass transfer, Chin. Phys. B, vol 22(2), pp. 024702(1-6), 2013.

[11] P. K. Kameswaran, Sshaw and P. Sibanda, Dual solutions of Casson fluid flow over a stretching or shrinking sheet, Sadhana, vol 39(6), pp. 1573-1583, 2014.

[12] N. Saidulu and A. V. Lakshmi, Slip Effects on MHD Flow of Casson Fluid over an Exponentially Stretching Sheet in Presence of Thermal Radiation, Heat Source/Sink and Chemical Reaction, Eur. J. Adv. Eng. Technol., vol 3(1), pp. 47-55, 2016.

[13] S. Shehzad, T. Hayat, M. Qasim and S. Asghar, Effects of mass transfer on mhd flow of casson fluid with chemical reaction and suction, Brazilian J. Chem. Eng., vol 30(1,) pp. 187-195, 2013.

[14] S. V. Jakati, R. B. T, A. L. Nargund and S. B. Sathyanarayana, Study of Casson nano MHD flow over a stretching sheet with non-uniform heat sourse/sink by HAM, Int. J. Mech. Eng. Technol., vol 9(10), pp. 931-945, 2018.

[15] J. Alinejad and S. Samarbakhsh, Viscous Flow over Nonlinearly Stretching Sheet with Effects of Viscous Dissipation, J. Appl. Math., 2012.

[16] C. Sumalatha and S. Bandari, Effects of Radiations and Heat Source/Sink on a Casson Fluid Flow over Nonlinear Stretching Sheet, World J. Mech., vol 5, pp. 257-265, 2015.

[17] M. Reza, R. Chahal, and N. Sharma, Radiation Effect on MHD Casson Fluid Flow over a Power-Law Stretching Sheet with Chemical Reaction,” Int. J. Chem. Mol. Eng., vol 10(5), pp. 585-590, 2016.

[18] S. Mukhopadhyay, Casson fluid flow and heat transfer over a nonlinearly stretching surface, Chin. Phys. B, vol 22(7), pp. 074701(1-5), 2013.

[19] M. Mustafa and J. A. Khan, Model for flow of Casson nanofluid past a non-linearly stretching sheet considering magnetic field effects, AIP Adv., vol 5, pp. 077148(1-11), 2015.

[20] I. Ullah, I. Khan and S. Shafie, MHD Natural Convection Flow of Casson Nanofluid over Nonlinearly Stretching Sheet Through Porous Medium with Chemical Reaction and Thermal Radiation, Nanoscale Res. Lett., 2016.

[21] I. Ullah, M. Qasim, I. Khan and S. Shafie, Heat and Mass Transfer Slip Flow of Casson Fluid over a Nonlinearly Stretching Sheet Saturated in a Porous Medium with Chemical Reaction, J.Math, 2017.

[22] R. I. Yahaya, N. M. Arifin and S. S. P. M. Isa, Stability Analysis on Magnetohydrodynamic Flow of Casson Fluid over a Shrinking Sheet with Homogeneous-Heterogeneous Reactions, Entropy, 2018.

[23] K. Vajravelu, Viscous Flow over a Nonlinearly Stretching Sheet, Appl. Math. Comput., vol 124, pp. 281-288, 2001.

[24] R. Cortell, Viscous Flow and Heat Transfer over a Nonlinearly Stretching Sheet, Appl. Math. Comput., vol. 184, pp. 864-873, 2007.