

Natural Convection Flow along a Vertical Wavy Cone with Uniform Surface Heat Flux and Temperature Dependent Viscosity

Azad Rahman¹, Md. Mamun Molla^{2,*} and Sharaban Thohura³

¹*School of Engineering & the Built Environment, Central Queensland University, Australia.*

²*Dept. of Mathematics & Physics, North South University, Dhaka-1229, Bangladesh.*

³*Dept. of Mathematics, Jagannath University, Dhaka-1000, Bangladesh.*

Abstract

The effect of temperature dependent viscosity $\mu(T)$, on steady two-dimensional natural convection flow along a vertical wavy cone with uniform surface heat flux, has been investigated numerically. Viscosity is considered to be a linear function of temperature T . Using the appropriate variables, the Navier-Stokes and energy equations are transformed into non-dimensional boundary layer equations and then solved numerically employing marching order implicit finite difference method with double sweep technique. The effects of viscosity variation parameter on the velocity profile, temperature profile, velocity vector field, skin friction coefficient, average Nusselt number, streamlines, and isotherm have been discussed by graphical representation.

Keywords: *Natural convection, Wavy cone, Variable viscosity, Heat flux, Finite Difference Method.*

Nomenclature

a	Amplitude wavelength ratio.	β	Volumetric coefficient of thermal expansion.
C_f	Skin friction coefficient.	E	Viscosity variation parameter.
Gr	Grashof number.	θ, Θ	Dimensionless temperature function.
k	Thermal conductivity.	μ	Viscosity of the fluid.
\hat{n}	Unit vector normal to the surface.	μ_∞	Dynamic viscosity of the ambient fluid.
Nu_m	Average Nusselt number.	ν_∞	Reference kinematic viscosity.
P	Dimensionless pressure function.	ρ	Density of the fluid.
Pr	Prandtl number.	$\sigma(x)$	Non-dimensional surface profile.
q_w	Uniform heat flux at the surface.	τ_w	Shearing stress.
$\hat{r}(\hat{x})$	Local radius of the of the cone.	Φ	The half angle of the cone.
r, R	Dimensionless radius of the cone.	ψ	Stream function.
T	Temperature in the boundary layer.		
T_∞	Temperature of the ambient fluid.		
T_w	Temperature at the surface.		
(u, v)	Dimensionless velocity component.		

Greek symbols

Subscripts

m	Average condition
∞	Ambient condition
x	Differentiation with respect to x

1. Introduction

Roughened surfaces are encountered in heat transfer devices such as flat plate solar collectors and flat plate condensers in refrigerators. Larger scale surface non-uniformities are encountered, for example, in cavity wall insulating systems and grain storage containers, room heater, etc. If the surface is wavy, the flow is disturbed by the surface, and this alters the rate of heat transfer.

The only papers to date that first study the effects of such non-uniformities on the vertical convective boundary layer flow of a Newtonian fluid are those of Yao [1], Moulic and Yao [2, 3]. The problem of free convection flow from a wavy vertical surface in the presence of a transverse magnetic field was studied by Alam et al. [4]. Natural convection over a vertical wavy cone and frustum of a cone has been studied by Pop and Na [5, 6]. Cheng [7] have investigated natural convection heat and mass transfer near a vertical wavy cone with constant wall temperature and concentration in a porous medium. Wang and Chen [8] have studied mixed convection boundary layer flow on inclined wavy plates including the magnetic field effect. Yao [9] studied natural convection along a vertical complex wavy surface. Molla et al. [10] studied the natural convection flow along a vertical complex wavy surface with uniform heat flux where the fluid viscosity is constant. Mutthy et al. [11] investigated the natural convection heat transfer from a horizontal wavy surface in a porous enclosure. Kumar [12] studied free convection induced by a vertical wavy surface with heat flux in a porous enclosure.

In all of the studies as mentioned above the viscosity of the fluids is constant in the flow regime. The physical properties of fluid may change significantly with temperature. For instance, the viscosity of water decreases about 240% when the temperature increases from 10°C to 50°C. Also, the viscosity of air is 0.6924×10⁻⁵ kg/m.s, 1.3289 kg/m.s, 2.286 kg/m.s and 3.625 kg/m.s at 1000K, 2000K, 4000K and 8000K temperature respectively [13].

To predict the flow behaviors accurately, it is necessary to take the viscosity into account. Gray et al. [14], Mehta and Sood [15] found that the flow characteristics substantially changed with the effect of temperature dependent viscosity. Ling and Dybbs [16] have considered the viscosity to vary inversely to the temperature which is appropriate for the fluid having large Prandtl number. On the other hand, Chrraudeau [17] has proposed a formula assuming the viscosity of the fluid to be proportional to a linear function of temperature. Following Chrraudeau [17], Hossain et al. [18-20] have studied the natural convection flow along a vertical wavy cone and wavy surface with uniform surface temperature in the presence of temperature dependent viscosity. Molla et al. [21] investigated the natural convection flow along a vertical wavy surface with temperature dependent viscosity and thermal conductivity. Very recently, Rahman et al. [22] investigated natural convection flow along the vertical wavy cone in case of uniform surface heat flux. They have considered viscosity to be an exponential function of temperature.

In many applications, the surface temperature is non-uniform. The case of uniform surface heat flux has great importance in engineering applications. In the present study, the free convection flow along a vertical wavy cone with surface heat flux and the viscosity is the linear function of temperature has been used which is appropriate for the small Prandtl number or gaseous fluid. The current problem is solved numerically using marching order implicit finite difference method. Solutions are obtained for the fluid having Prandtl number $Pr = 0.7$ (air) with the different values of viscosity variation parameter. Also, the effects of the amplitude of the waviness on the solution are observed.

2. Formulation of the Problem

The boundary layer analysis outlined below allows the shape of the wavy surface, $\hat{\sigma}(\hat{x})$ to be arbitrary, but our detailed numerical work will assume that the surface exhibits sinusoidal deformations. Thus, the wavy surface of the cone is described by

$$\hat{y}_w = \sigma(\hat{x}) = \hat{a} \sin(\pi \hat{x}/L) \quad (1)$$

where $2L$ is the fundamental wavelength associated with wavy surface and \hat{a} is the amplitude of

the waviness.

The physical model of the problem and the two-dimensional coordinate system are shown in Figure 1, where φ is the half angle of the flat surface of the cone and $\hat{r}(\hat{x})$ is the local radius of the flat surface of the cone which is defined by

$$\hat{r} = \hat{x} \sin \varphi \quad (2)$$

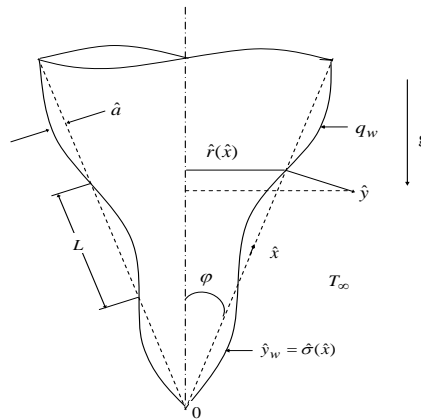


Figure 1. Physical model and the coordinate system.

Under the Boussinesq approximation, we consider the flow to be governed by the following equations:

$$\frac{\partial(\hat{r}\hat{u})}{\partial\hat{x}} + \frac{\partial(\hat{r}\hat{v})}{\partial\hat{y}} = 0 \quad (3)$$

$$\hat{u} \frac{\partial\hat{u}}{\partial\hat{x}} + \hat{v} \frac{\partial\hat{u}}{\partial\hat{y}} = -\frac{1}{\rho} \frac{\partial\hat{p}}{\partial\hat{x}} + \frac{1}{\rho} \bar{\nabla} \cdot (\mu \bar{\nabla} \hat{u}) + g\beta(T - T_\infty) \cos\varphi \quad (4)$$

$$\hat{u} \frac{\partial\hat{v}}{\partial\hat{x}} + \hat{v} \frac{\partial\hat{v}}{\partial\hat{y}} = -\frac{1}{\rho} \frac{\partial\hat{p}}{\partial\hat{y}} + \frac{1}{\rho} \bar{\nabla} \cdot (\mu \bar{\nabla} \hat{v}) + g\beta(T - T_\infty) \sin\varphi \quad (5)$$

$$\hat{u} \frac{\partial T}{\partial\hat{x}} + \hat{v} \frac{\partial T}{\partial\hat{y}} = \frac{k}{\rho c_p} \nabla^2 T \quad (6)$$

where (\hat{x}, \hat{y}) are the dimensional coordinates and (\hat{u}, \hat{v}) are the velocity components parallel to (\hat{x}, \hat{y}) . Also k is the thermal conductivity, C_p is the specific heat at constant pressure and μ is the temperature dependent viscosity of the fluid which is defined as a linear function of the temperature.

$$\mu = \mu_\infty [1 + \gamma(T - T_\infty)] \quad (7)$$

where μ_∞ is the viscosity of ambient fluid outside the boundary layer and γ is a constant.

The boundary condition for the present problem is

$$\hat{u} = 0, \hat{v} = 0, q_w = -k(\hat{n} \cdot \bar{\nabla} T) \text{ at } \hat{y} = \hat{y}_w = \sigma(\hat{x}) \quad (8a)$$

$$\hat{u} = 0, T = T_\infty \text{ as } \hat{y} \rightarrow \infty \quad (8b)$$

where q_w is the uniform heat flux and \hat{n} is the unit vector normal to the wavy surface.

Now the following non-dimensional variables are introduced to obtain a set of non-dimensional governing equation:

$$x = \frac{\hat{x}}{L}, y = \frac{\hat{y} - \sigma(\hat{x})}{L} Gr^{1/5}, r = \frac{\hat{r}}{L}, a = \frac{\hat{a}}{L}, \sigma(x) = \frac{\sigma(\hat{x})}{L}, \sigma_x = \frac{d\sigma}{d\hat{x}} = \frac{d\sigma}{dx}, p = \frac{L^2}{\rho v_\infty^2} Gr^{-4/5} \hat{p}, u = \frac{\rho L}{\mu_\infty} Gr^{-2/5} \hat{u}, v = \frac{\rho L}{\mu_\infty} Gr^{-1/5} (\hat{v} - \sigma_x \hat{u}), \theta = \frac{T - T_\infty}{(q_w L / k)} Gr^{1/5}, Gr = \frac{g\beta q_w \cos\varphi}{(k v_\infty^2)} L^4 \quad (9)$$

where θ is the dimensionless temperature function and $\nu_\infty = \mu_\infty / \rho$ is the kinematic viscosity. Here the new coordinate system (x, y) are not orthogonal, but a regular rectangular computational grid

can be easily fitted in the transformed coordinate. On introducing the above dimensionless dependent and independent variables into the equations (3)-(6) the following dimensionless form of the governing equations are obtained at leading order in the Grashof number, $Gr \rightarrow \infty$:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \quad (10)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \sigma_x Gr^{1/5} \frac{\partial p}{\partial y} + \varepsilon(1 + \sigma_x^2) \frac{\partial \theta}{\partial y} \frac{\partial u}{\partial y} + (1 + \varepsilon\theta)(1 + \sigma_x^2) \frac{\partial^2 u}{\partial y^2} + \theta \quad (11)$$

$$\sigma_x \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \sigma_{xx} u^2 = -Gr^{1/5} \frac{\partial p}{\partial y} + \sigma_x(1 + \varepsilon\theta)(1 + \sigma_x^2) \frac{\partial^2 u}{\partial y^2} + \varepsilon\sigma_x(1 + \sigma_x^2) \frac{\partial \theta}{\partial y} \frac{\partial u}{\partial y} - \theta \tan \varphi \quad (12)$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} (1 + \sigma_x^2) \frac{\partial^2 \theta}{\partial y^2} \quad (13)$$

where

$$Pr = \frac{\mu_\infty c_p}{k} \quad \text{and} \quad \varepsilon = \gamma \frac{q_w L}{k} Gr^{-1/5} \quad (14)$$

It can easily be seen that the convection induced by the wavy surface is described by equations (10)-(13). Equation (12) represents that the pressure gradient along the x direction is in the order of $Gr^{-1/5}$. In the present problem this pressure gradient is zero because, no externally induced free stream exists. The elimination of $\partial p / \partial y$ from equations (11) and (12) leads to

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = (1 + \varepsilon\theta)(1 + \sigma_x^2) \frac{\partial^2 u}{\partial y^2} + \varepsilon(1 + \sigma_x^2) \frac{\partial \theta}{\partial y} \frac{\partial u}{\partial y} - \frac{\sigma_x \sigma_{xx}}{1 + \sigma_x^2} u^2 + \frac{(1 - \sigma_x \tan \varphi)}{1 + \sigma_x^2} \theta \quad (15)$$

The corresponding boundary conditions for the present problem then turn into

$$u = 0, \quad v = 0, \quad \frac{\partial \theta}{\partial y} = -\frac{1}{\sqrt{1 + \sigma_x^2}} \quad \text{at} \quad y = 0 \quad (16a)$$

$$u = 0, \quad \theta = 0 \quad \text{as} \quad y \rightarrow \infty \quad (16b)$$

Now, we introduce the following transformations to reduce the governing equation to a convenient form

$$X = x, \quad Y = \frac{y}{(5x)^{1/5}}, \quad R = r, \quad U(X, Y) = \frac{u}{(5x)^{3/5}} \quad (17a)$$

$$V(X, Y) = (5x)^{1/5} v, \quad \Theta(X, Y) = \frac{\theta}{(5x)^{3/5}} \quad (17b)$$

Introducing the transformations given in equation (17) into the equations (10), (15) and (13), we have

$$(5X) \frac{\partial U}{\partial X} - Y \frac{\partial U}{\partial Y} + 8U + \frac{\partial V}{\partial Y} = 0 \quad (18)$$

$$(5X)U \frac{\partial U}{\partial X} + (V - YU) \frac{\partial U}{\partial Y} + \left[3 + \frac{\sigma_x \sigma_{xx}}{1 + \sigma_x^2} (5X) \right] U^2 = \left[1 + \varepsilon(5X)^{1/5} \Theta \right] (1 + \sigma_x^2) \frac{\partial^2 U}{\partial Y^2} + (5X)^{1/5} \varepsilon (1 + \sigma_x^2) \frac{\partial \Theta}{\partial Y} \cdot \frac{\partial U}{\partial Y} + \frac{(1 - \sigma_x \tan \varphi)}{1 + \sigma_x^2} \Theta \quad (19)$$

$$(5X)U \frac{\partial \Theta}{\partial X} + (V - YU) \frac{\partial \Theta}{\partial Y} + U\Theta = \frac{1}{Pr} (1 + \sigma_x^2) \frac{\partial^2 \Theta}{\partial Y^2} \quad (20)$$

The boundary conditions (16) now take the following form:

$$U = 0, \quad V = 0, \quad \frac{\partial \Theta}{\partial Y} = -\frac{1}{\sqrt{1 + \sigma_x^2}} \quad \text{at} \quad Y = 0 \quad (21a)$$

$$U = 0, \quad \Theta = 0 \quad \text{as} \quad Y \rightarrow \infty \quad (21b)$$

Solutions of the system of partial differential equations are obtained using the marching order implicit finite difference method. Equations (18)-(20) are discretized for numerical scheme using central difference for the diffusion term and backward difference for the convection terms. Finally, we get a system of tri-diagonal algebraic equations which was solved by Gaussian elimination method. The computation is started at $X = 0.0$, and then marches up to the point $X = 10.0$. Here,

$\Delta X = 0.005$ and $\Delta Y = 0.01$ are used for the X and Y grids respectively.

However, once we know the values of the function U , V and Θ and their derivatives, it is important to calculate the values of the average Nusselt number, Nu_m from the following relation:

$$Nu_m(5/Gr)^{1/5} = \frac{x^{1/5} \int_0^X \sqrt{1+\sigma_x^2} dx}{\int_0^X \sqrt{1+\sigma_x^2} x^{1/5} \Theta(x,0) dx} \quad (22)$$

Also the skin friction coefficient is defined as

$$C_{fx}(Gr)^{1/5}/\{2(5x)^{2/5}\} = (1 + \varepsilon(5x)^{1/5}\Theta)\sqrt{(1 + \sigma_x^2)} \left. \frac{\partial U}{\partial y} \right|_{y=0} \quad (23)$$

The stream function for the wavy cone is defined as

$$u = \frac{1}{r} \frac{\partial \psi}{\partial y}, \text{ and } v = -\frac{1}{r} \frac{\partial \psi}{\partial x} \quad (24)$$

For calculating the stream function ψ , we have integrated the fluid velocity over the whole boundary layer, which may be defined as

$$\psi = \int_0^Y R(5x)^{3/5} U dY, \quad \text{where } R = X \sin \varphi \quad (25)$$

3. Results and Discussion

In this paper, the effect of temperature dependent viscosity on a steady two-dimensional natural convection laminar flow of viscous incompressible fluid along a vertical wavy cone has been investigated using very efficient marching order finite difference method. It is seen that the solutions are affected by the viscosity variation parameter, ε , as well as the amplitude of the waviness, a , keeping the angle of the half cone $\varphi = 30^\circ$ is fixed. Here our attention is focused on the effect of ε and a on the average Nusselt number $Nu_m(5/Gr)^{1/5}$, skin friction coefficient C_{fx} as well as velocity and temperature distribution fluid of the. We also show the graphical representation of velocity vectors, streamlines and isotherms of the flow field.

To validate the present numerical results the skin friction coefficient and the surface temperature have been compared with those of Lin [23] and Pullepu et al. [24]. The detail of the comparison has been given in Rahman et al. [21]. The present result shows very good agreement with the results mentioned above.

The numerical results are presented for the different values of viscosity variation parameter ε and amplitude wavelength ratio a for a suitable fluid having Prandtl number $Pr = 0.7$. Firstly, to examine the effect of ε we have considered that $a = 0.3$ and $\varphi = 30^\circ$ remain constant. Figure 2(a-b) represents the non-dimensional tangential and normal velocity distribution for different values of ε at a fixed point $x = 1.0$. From the tangential velocity distribution, it is found that the increasing value of ε decreases the tangential velocity inside the boundary layer. As viscosity is a linear function of temperature, increasing value of ε indicate rapid change of viscosity towards upstream, which causes the decrease of fluid velocity. Figure 2(b) shows that the normal velocity enhances with ε increases.

Figure 3 (a) illustrates the temperature distribution for different values of ε at a fixed point $x = 1.0$. It is evident from the figure that the temperature inside the boundary layer at any fixed point raises with ε . From the surface temperature profile on Figure 3(b), it is found that the wall temperature increases significantly due to the increasing value of ε . This is not surprising as the viscosity is a linear function of temperature.

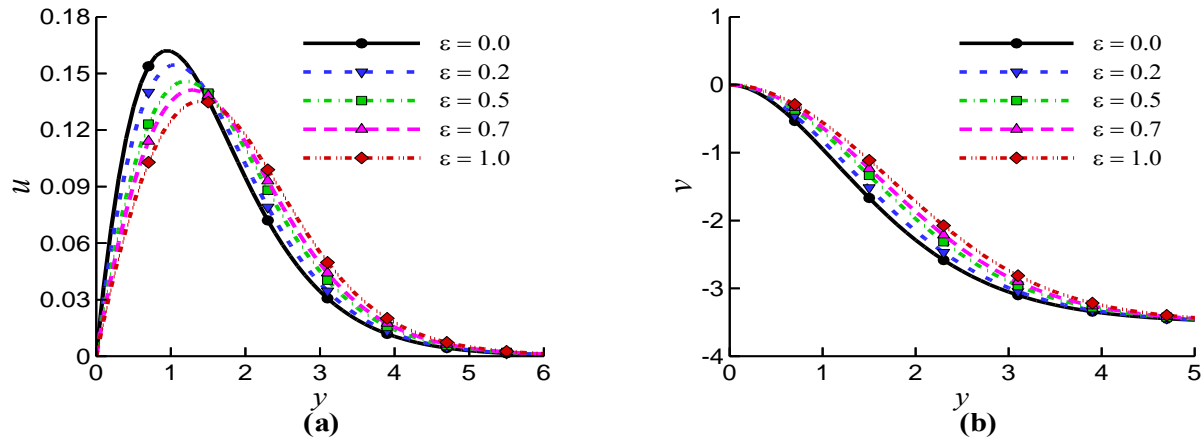


Figure 2. (a) Tangential velocity distribution and (b) Normal velocity distribution at $X=1.0$ for Prandtl number $Pr = 0.7$, $a = 0.3$ and $\varphi = 30^\circ$.

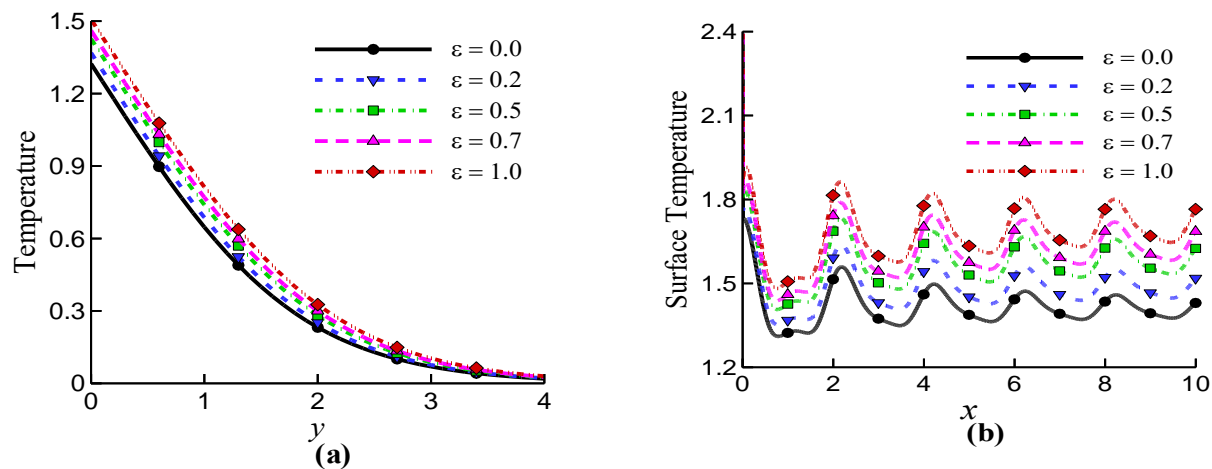


Figure 3. (a) Fluid temperature distribution at $X=1.0$ and (b) Surface Temperature distribution for $Pr = 0.7$, $a = 0.3$ and $\varphi = 30^\circ$.

The effects of ϵ on the skin friction coefficient and on the average rate of heat transfer are given in the Figure 4(a-b) respectively. The skin-friction coefficient increases throughout the computational domain for increasing value of ϵ . Due to the increase of ϵ , the viscosity increases with temperature which results the increase of skin friction. Also, we have found that the amplitude of the skin friction distribution enhanced with ϵ . While the average rate of heat transfer goes down when ϵ increases. It is to be mentioned that the complete cycle of the wavy surface is from $x = 0.0$ to $x = 2$. The skin-friction coefficient increases for the first quarter of the cycle ($x \approx 0$ to $x \approx 0.5$) and decreases in the second quarter ($x \approx 0.5$ to $x \approx 1$). From $x \approx 1.0$ to $x \approx 1.5$ (i.e. third quarter) skin-friction again increase, whereas in last quarter ($x \approx 1.5$ to $x \approx 2$) it decreases. The skin-friction coefficient showed the similar characteristic throughout the domain.

Figure 5 (a-c) shows the isotherm for a wavy cone, while the viscosity variation parameter ϵ are taken as 0.0, 0.5 and 1.0 respectively. These figures indicate that with the increase of ϵ , the thickness of thermal boundary layer increases slightly.

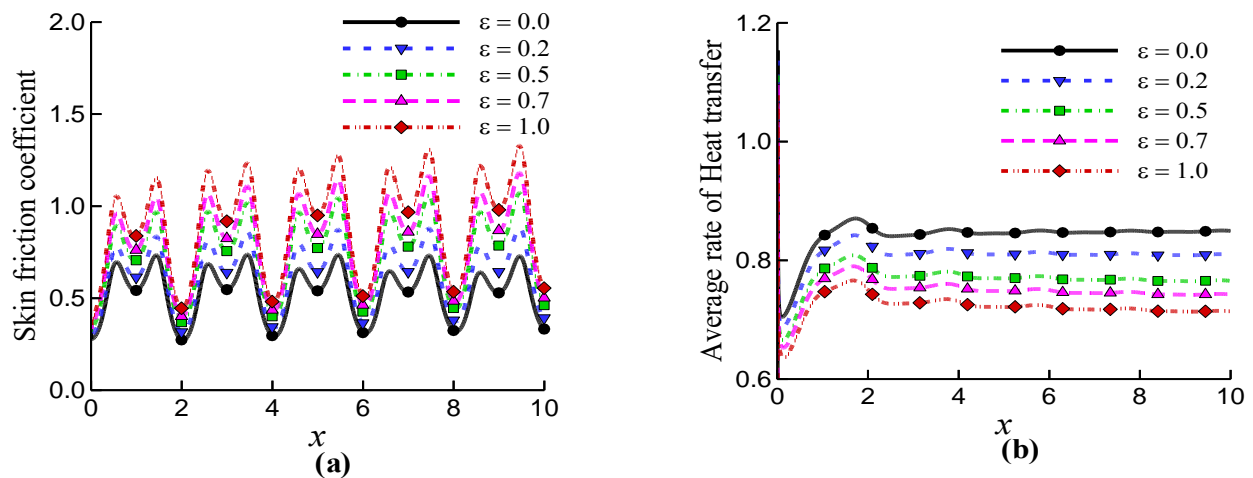


Figure 4. (a) Skin friction coefficient and (b) Average rate of heat transfer for $Pr = 0.7$, $a = 0.3$ and $\phi = 30^\circ$.

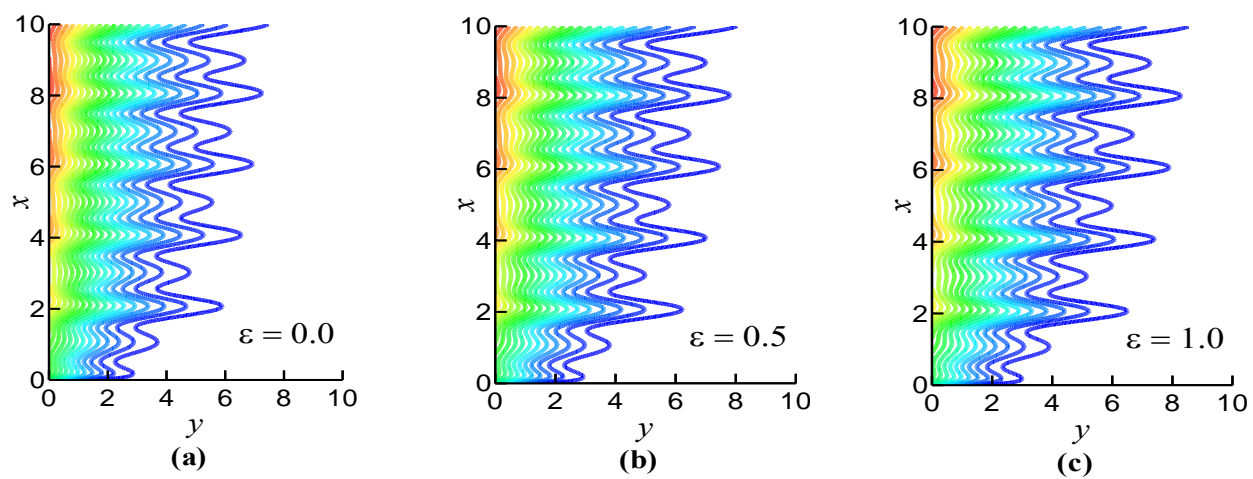


Figure 5. Isotherm for different values of ε for a wavy cone with $a = 0.3$, $\phi = 30^\circ$ and $Pr = 0.7$.

Velocity vectors for three different values of ε are shown in Figure 6, where the length of the vector is taken relative to their magnitudes. Velocity vectors took the form as shown in Figure.6 (a) when the viscosity remains constant on the entire domain. Velocity vector shows slightly disturbance when the value of ε increased since it affects the flow pattern with increasing the temperature. It is clear from the figure that perturbed vector fields enhance as viscosity increases.

Variation due to the amplitude of the wave surface has also been studied numerically considering viscosity variation parameter $\varepsilon = 0.5$ and half angle of the cone $\phi = 30^\circ$. Figure 7 demonstrate the effect of the amplitude on the normal and tangential velocity distribution at $x = 1.0$. From the figure it is clear that the tangential velocity at any point for wavy cone is smaller than that for a flat cone. The increasing value of the amplitude of the wavy surface retards the fluid motion near the surface but generate more fluid motion away from the surface resulting in a much thicker momentum boundary layer. It was found that for the wavy cone with high amplitude ($a = 0.5$) the velocity

reaches to its maximum far away from the surface comparing the case for flat cone. The normal velocity also reduces significantly with the increasing value of the amplitude.

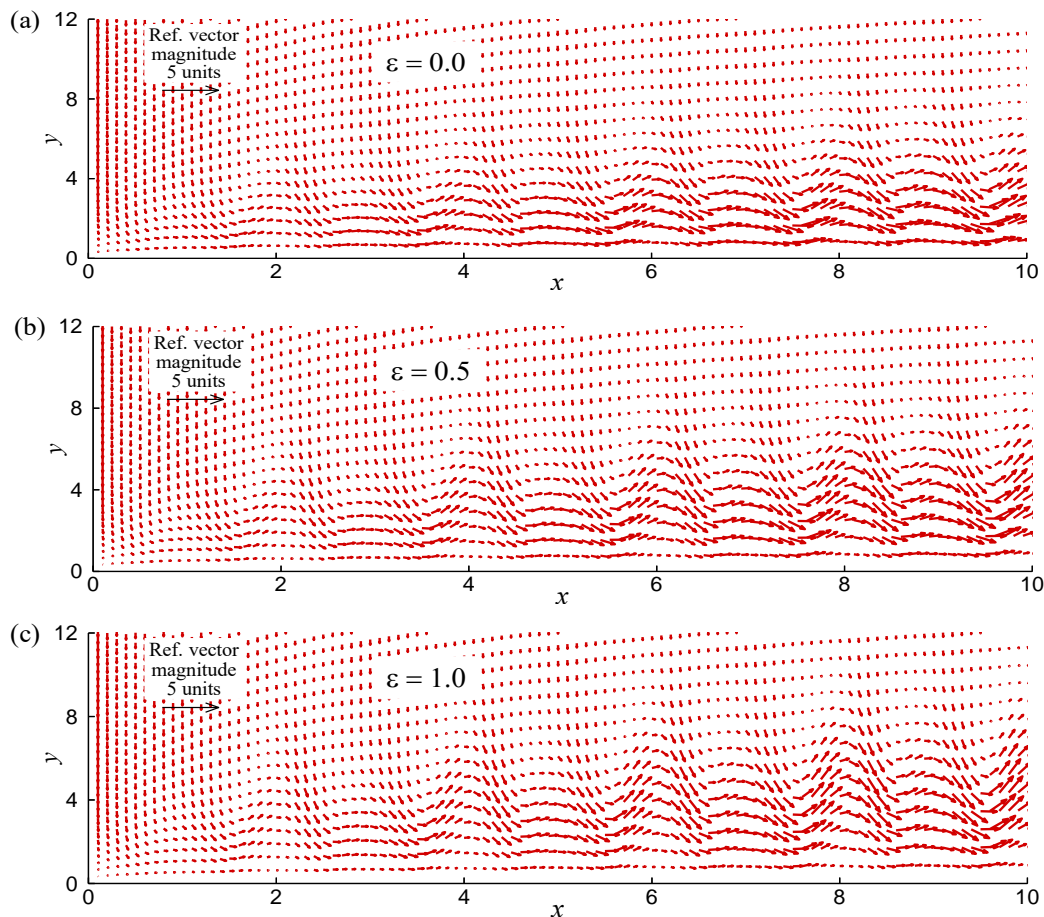


Figure 6. Velocity vectors for different values of ϵ for a wavy cone with $a = 0.3$, $\varphi = 30^\circ$ and $Pr = 0.7$.

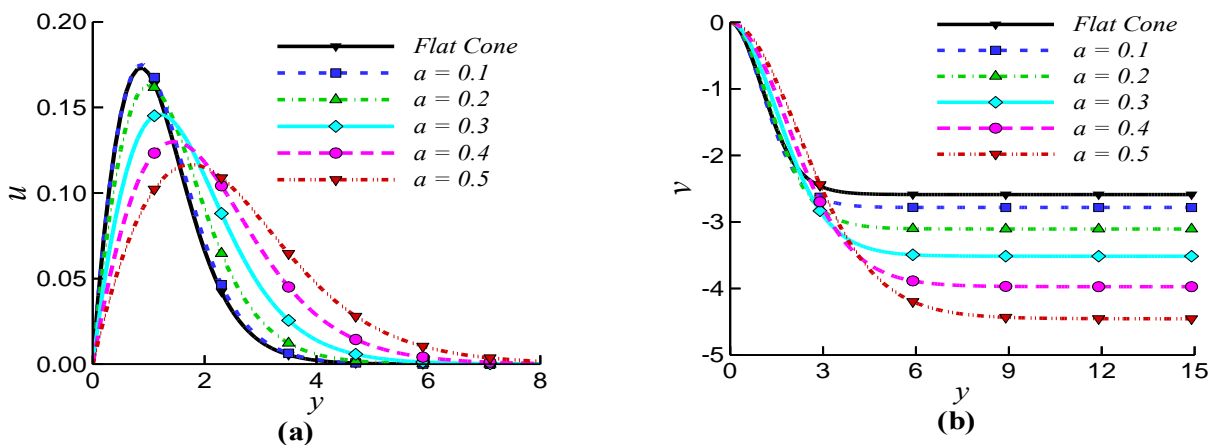


Figure 7. (a) Tangential velocity distribution and (b) Normal velocity distribution at $X=1.0$ for Prandtl number $Pr = 0.7$, $\epsilon = 0.5$ and $\varphi = 30^\circ$.

Figure 8(a) represents the fluid temperature distribution at $x = 1.0$ for different values of waviness

amplitude. It shows that the fluid temperature distribution is more sensitive to higher value of the amplitude. For lower value of the amplitude, temperature profile shows sharp decrease compared to the larger value of a . The effect of amplitude of the wavy surface on the surface shear stress in terms of the skin friction coefficient is given in the Figure 8(b). It is seen that the skin friction coefficient exhibits a sinusoidal behavior along the wavy surface. At every crest and trough, skin friction coefficient increases with the increasing value of a . But at the point of inflexion of the wavy surface, skin friction coefficient reduced significantly for a wavy surface with high amplitude. When the surface of the cone is not flat ($a \neq 0$) the component of the buoyancy force along the cone is reduced by the factor $(1 - \sigma_x \tan \phi) / (1 + \sigma_x^2)$, as shown in equation (19) from its maximum value of a flat cone. Consequently, the rate of skin friction and rate of heat transfer are reduced.

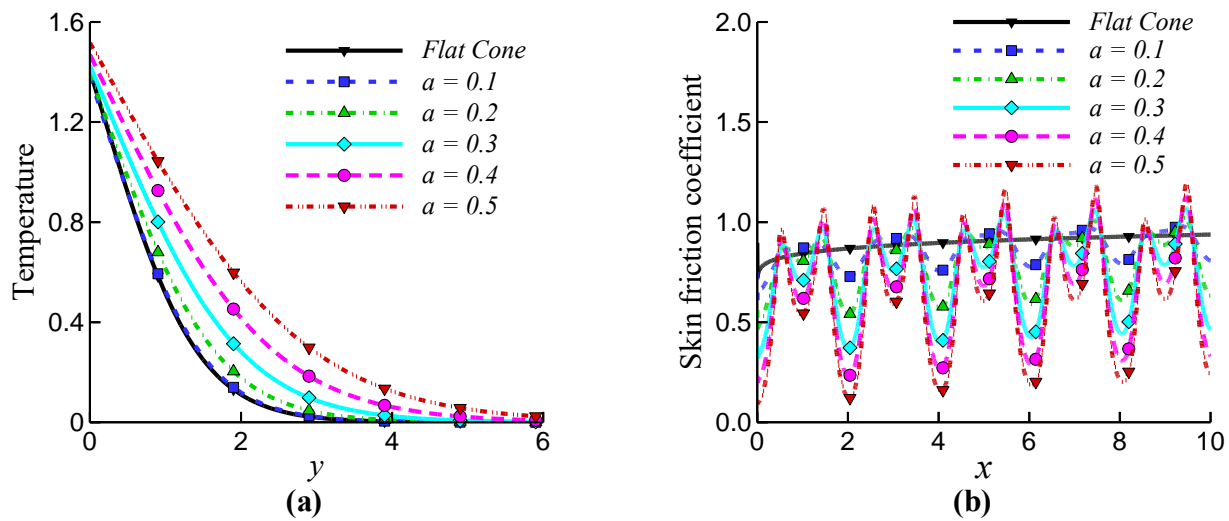


Figure 8. (a) Fluid temperature distribution at $X=1.0$ and (b) Skin friction coefficient for $Pr = 0.7$, $\varepsilon = 0.5$ and $\phi = 30^\circ$.

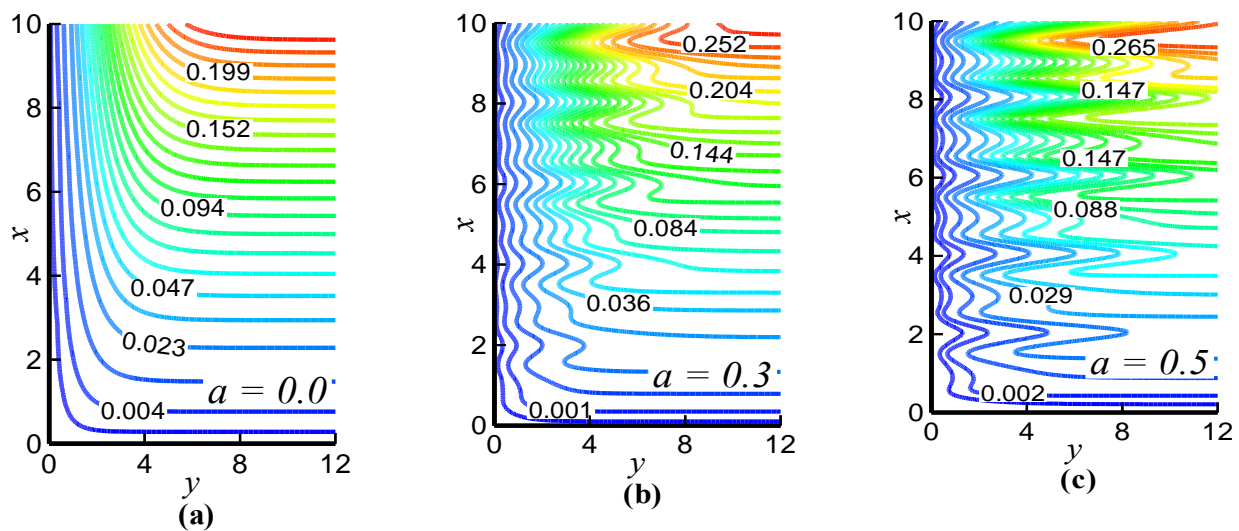


Figure 9. Streamlines for different values of a where $Pr = 0.7$, $\varepsilon = 0.5$ and $\phi = 30^\circ$.

Streamline pattern for a flat cone and wavy cone with amplitude $a = 0.3$ and $a = 0.5$ are illustrated in the Figure 9, while Figure 10 shows the effect of surface amplitude on the isotherms. Within the computational domain for flat cone, the maximum value of stream function $\Psi_{max} = 0.246$. While for a wavy cone with amplitude $a = 0.3$ and $a = 0.5$, Ψ_{max} is 0.276 and 0.309, respectively. Which indicate that the value of stream function enhances with the amplitude of the wavy cone and causing thicker momentum and thermal boundary layer. It is also found that unlike to the flat cone, the isotherm for wavy cone shows a sinusoidal behavior. These figures clearly show that the thickness of thermal boundary layer increases significantly as the amplitude of the wavy surface increases.

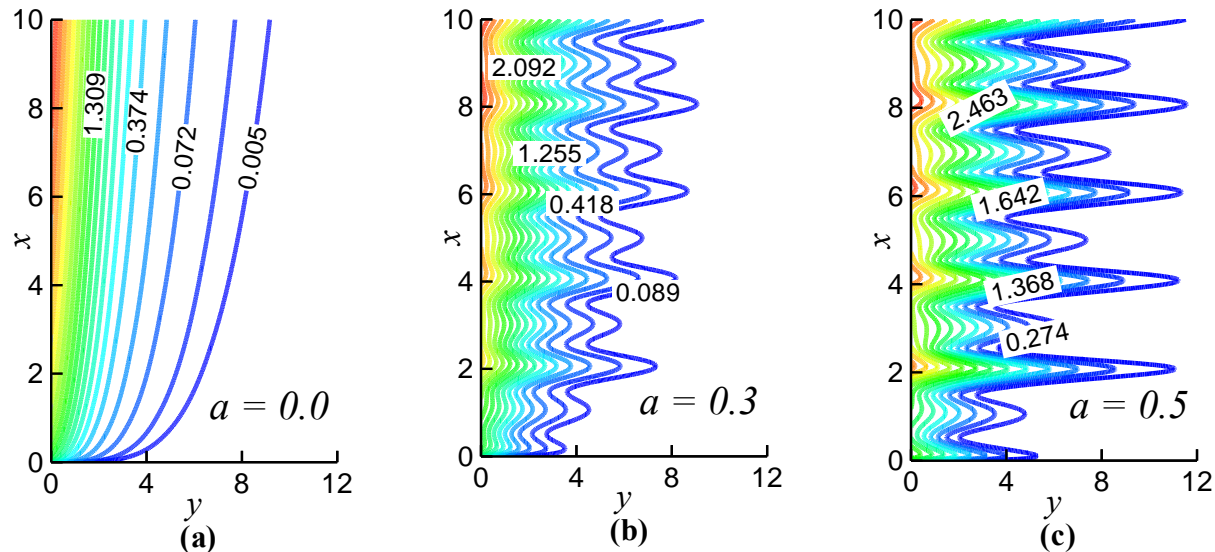


Figure 10. Isotherms for different values of a where $Pr = 0.7$, $\varepsilon = 0.5$ and $\phi = 30^\circ$.

4. Conclusions

The effect of the temperature-dependent viscosity on the natural convection boundary layer flow along a vertical wavy surface with uniform heat flux has been studied numerically. New variables transform the complex geometry into a simple shape where a very efficient marching order implicit finite difference method was used to solve the non-dimensional boundary layer equations. From the present investigation the result can be summarized as follows.

- The skin friction increases within the computational domain for increasing value of the viscosity variation parameter ε .
- For a wavy cone at every crest and trough, skin friction increases with the increasing value of amplitude a . At the point of inflexion the skin friction coefficient reduce significantly for a wavy surface with high amplitude.
- The average rate of heat transfer decreases significantly for increasing value of viscosity variation parameter.
- Tangential velocity reduces with the increasing value of viscosity variation parameter ε . Similar phenomenon was noted with the increase of amplitude of the surface waviness.
- Fluid temperature and surface temperature were found to enhance with ε .
- One important finding is that, with the higher value of ε , the thickness of thermal boundary layer increases slightly.

- It was found that, streamlines change significantly with the increasing value of amplitude. Also, the maximum value of stream function enhances with increases of the waviness amplitude

The results show that without considering viscous effect may introduce sever error in the prediction of the surface rate of heat transfer and skin friction coefficient.

References

- [1] L. S. Yao, Natural convection along a vertical wavy surface, ASME J. Heat Transfer vol. 105, pp. 465-468, 1983.
- [2] S. G. Moulic and L. S. Yao, Natural convection along a wavy surface with uniform heat flux, ASME J. Heat Transfer vol. 111, pp.1106-1108, 1989.
- [3] S. G. Moulic and L. S. Yao, Mixed convection along wavy surface, ASME J. Heat Transfer vol. 111, pp. 974-979, 1989.
- [4] K. C. A. Alam, M. A. Hossain and D. A. S. Ree, Magnetohydrodynamic free convection along a vertical wavy surface, Appl. Mech. Engrg. vol. 1, pp. 555-566, 1997.
- [5] I. Pop and T. Y. Na, Natural convection from a wavy cone, applied Scin. Res. vol. 54, pp. 125-136, 1995.
- [6] I. Pop and T. Y. Na, Natural convection over a vertical wavy frustum of a cone, Int. J. Non-Linear Mech. vol. 34, pp. 925-934, 1999.
- [7] C. Y. Cheng, Natural convection heat and mass transfer near a vertical wavy cone with constant wall temperature and concentration in a porous medium, Mech. Res. Commun., vol. 27, pp. 613-620, 2000.
- [8] C. C. Wang, and C. K. Chen, Mixed convection boundary layer flow on inclined wavy plates including the magnetic field effect, Int. J. Therm. Sci., vol. 44, pp. 577-586, 2005.
- [9] L. S. Yao, Natural convection along a vertical complex wavy surface, Int. J. Heat Mass Transfer, vol. 49, pp.281-286, 2006.
- [10] M. M. Molla, M. A. Hossain and L. S. Yao, Natural convection flow along a vertical complex wavy surface with uniform heat flux, ASME J. Heat Transfer vol. 129 (10), pp. 1403-1407, 2007.
- [11] P.V.S.N. Murthy, B.V. R. Kumar and P. Singh, Natural Convection Heat Transfer from a horizontal wavy surface in a porous enclosure, Numerical Heat Transfer, Part A, vol. 31 (2), pp. 207-221, 1997.
- [12] B. V. R. Kumar, A study of free convection induced by a vertical wavy surface with heat flux in a porous enclosure, Numerical Heat Transfer, Part A, vol. 37 (1-8), pp. 493-510, 2000.
- [13] T. Cebeci and P. Bradshaw, Physical and Computational Aspects of Convective heat Transfer, Springer, New York, 1984.
- [14] J. Gray, D. R. Kassory and H. Tadjeran, The effect of significant viscosity variation on convective heat transport in water- saturated porous medium, J. Fluid Mech. vol. 117, pp. 233-249, 1982.
- [15] K. N. Mehta and S. Sood, Transient free convection flow with temperature dependent viscosity in fluid saturated porous media, Int. J. Engrg Sci., vol. 30, pp. 1083-1087, 1992.
- [16] J. X. Ling and A. Dybbs, Forced convection over a flat plate submersed in a porous medium: variable viscosity case, Paper 87 -WA/HT-23, ASME, New York, 1987.
- [17] J. Chraudeau, Influence de gradients de proprietes physiques en convection force application au cas du tube, Int. J. Heat Mass Tran. vol. 18, pp 87-95, 1975.
- [18] M. A. Hossain, M. S. Munir, and I. Pop, Natural convection flow of viscous fluid with viscosity inversely proportional to linear function of temperature from a vertical cone, Int. J. Therm. Sci. vol. 40,

pp. 366-371, 2001.

- [19] M. A. Hossain, M. S. Munir and I. Pop, Natural convection of a viscous fluid with variable viscosity and thermal conductivity from a vertical wavy cone, *Int. J. Therm. Sci.*, vol. 40, pp. 437-443, 2001.
- [20] M. A. Hossain, M. S. Munir and D. A. S. Rees, Flow of viscous incompressible fluid with temperature dependent viscosity and thermal conductivity past a permeable wedge with uniform surface heat flux, *Int. J. Therm. Sci.*, vol. 39, pp. 635-644, 2000.
- [21] A. Rahman, M.M. Molla and M. M. A. Sarker, Natural convection flow along the vertical wavy cone in case of uniform surface heat flux where viscosity is an exponential function of temperature, *Int. Comm. Heat and Mass Transfer*, vol. 38 (6), pp 774-780, 2011.
- [22] F. N. Lin, Laminar convection from a vertical cone with uniform surface heat flux, *Letters in Heat and Mass Transfer*, vol. 3, pp. 49-58, 1976.
- [23] B. Pullepu, K. Ekambavanan, and A. J. Chamkha, Unsteady laminar free convection from a vertical cone with uniform surface heat flux, *Nonlinear Analysis: Modeling & Control*, vol. 13(1), pp. 47-60, 2008.

Biographical information

Azad Rahman, PhD

School of Engineering & Built Environment
Central Queensland University,
Australia.



He completed his Ph.D from school of Engineering & Built Environment, Central Queensland University, Australia. Dr. Rahman has a strong academic background and research experience in the field of CFD. He graduated in 1998 in Mathematics from Dhaka University. He also completed his M.Sc. program from the same university with dissertation as a part of the degree. In his M.Sc. dissertation he worked with nonlinear system of equation and solved those by using FORTRAN. Later on he completed his M.Phil program from Bangladesh University of Engineering and Technology (BUET). After completing his MSc, he joined in Stamford University, Bangladesh as a Lecturer in Mathematics in 2002. In 2006, he joined as Assistant Professor in the same Department. During his M.Phil program he was involved with the research work based on CFD. He wrote several technical papers that he worked for. He attended several international conferences and presented his research work. He has several peer reviewed publications in

different journals and conferences.

Md. Mamun Molla, PhD

Associate Professor
Department of Mathematics & Physics,
North South University, Dhaka-
1229, Bangladesh



He received his BSc (Honors) in Mathematics and MSc in Applied Mathematics from the Department of Mathematics, University of Dhaka, Bangladesh. He did MPhil in Applied Mathematics, from the Department of Mathematics, Bangladesh University of Engineering & Technology (BUET). After completing his MSc, he joined in BUET as a visiting Lecturer in Mathematics and in 2004 n he joined in BUET as a full-time lecturer. In 2005, he was awarded the Overseas Research Scholarship (ORS) by the UK government for doing his PhD in Computational Fluid Dynamics (CFD) in the Department of Mechanical Engineering, University of Glasgow, UK and he completed his PhD in July 2009. After completing his PhD, from October 2009 to August 2011 he performed as a Postdoctoral Research Associate in the field of CFD using high performance parallel computing in the Department of Mechanical

and Manufacturing Engineering, University of Manitoba, Canada. Now Dr. Molla is an Associate Professor in the Department of Mathematics & Physics, North South University, Bangladesh.

His current research interests are: Boundary-layer flow, turbulent flow using Direct Numerical Simulation (DNS), Large-Eddy Simulation (LES), hybrid RANS/LES technique, Natural and Mixed convection flow, Lattice Boltzmann Method, Computational blood-flow as well as high performance parallel computing using MPI and OpenMP based CPU as well as GPU computing. He has over 50 peer-reviewed international journal publications and over 20 international conference papers. He attended different international conferences and seminars in home and abroad.

Sharaban Thohura, PhD

Assistant Professor, Department of Mathematics
Jagannath University, Dhaka-1100, Bangladesh.

She is an Assistant Professor in the Department of Mathematics at Jagannath University, Bangladesh. She was enrolled in Ph.D. program in October, 2014 at the



Department of Mathematics, Bangladesh University of Engineering & Technology (BUET) and have been continuing her research to till date. In 2016, She was awarded the NST fellowship by the Ministry of Science & Technology, Bangladesh for doing her Ph.D. She received her B.Sc. (Honors) in Mathematics and M.S. in Applied Mathematics from the Department of Mathematics, University of Dhaka, Bangladesh. She did MPhil. in Applied Mathematics, from the Department of Mathematics, Bangladesh University of Engineering & Technology (BUET). After completing his MSc, she joined in Stamford University, Bangladesh as a Lecturer in Mathematics in 2005. In 2011, she joined as a lecturer in the Department of Mathematics at Jagannath University, Bangladesh. Her current research interests include computational fluid Dynamic (CFD), Boundary-layer flow, Natural and Mixed convection flow, Numerical simulation of non-Newtonian fluid flow and Numerical Analysis. She has 6 peer-reviewed international journal publications and 7 international conference papers. She attended different international conferences and seminars in home.