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Boundary layer flow and heat transfer of a Casson fluid over a stretching sheet with Newtonian heating

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Abstract

This investigation focused on the case of steady boundary layer flow and heat transfer of non-Newtonian fluid over a stretching sheet with Newtonian heating (NH), in which heat transfer from the surface is proportional to the local surface temperature. Casson fluid model is used to characterize the non-Newtonian fluid behavior. The transformed governing nonlinear boundary layer equations are solved numerically by means of the very robust computer algebra software MATLAB employing the routine bvpc45. Numerical solutions are obtained for heat transfer from the stretching sheet and the wall temperature for a Casson parameter and a large range of values of Prandtl number Pr. The Newtonian heating is controlled by a dimensionless conjugate parameter, which varies between zero (insulated wall) and infinity (wall temperature remains constant). The important findings in this study are the variation of the surface temperature and heat flux from the stretching surface with the Casson parameter β, the conjugate parameter γ and Prandtl number Pr. It is observed that these parameters have essential effects on the heat transfer characteristics. A comprehensive numerical computation is carried out for various values of the parameters that describe the flow characteristics, and the results are reported graphically.

Keywords: *Stretching sheet; Casson fluid; boundary layer, Newtonian heating*.

Nomenclature

a	Positive constant
f	Dimensionless stream function
$\tilde{h}$	Heat transfer parameter
k	Thermal conductivity
Nu	Nusselt number
Pr	Prandtl number
q_w	Surface heat flux
Re	Local Reynolds number
T	Temperature of the fluid
(u, v)	Velocity components of the fluid
u_w	Velocity of stretching sheet
(x, y)	coordinate axes

Greek symbols

α	Thermal diffusivity
β	Casson parameter
ψ	Stream function
ν	Kinematic viscosity
γ	Conjugate parameter
θ	Dimensionless temperature
η	Similarity variable

Subscripts

ω	Conditions at the surface of cylinder
∞	Conditions in the free stream

1. Introduction

The situation with Newtonian heating arises in what are usually termed conjugate convective flows, where the heat is supplied to the convective fluid through a bounding surface with a finite heat capacity. This configuration occurs in many important engineering devices, for example in heat exchanger where the conduction in solid tube wall is greatly influenced by the convection in

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the fluid flowing over it. Further, for conjugate heat transfer around fins where the conduction within the fin and the convection in the fluid surrounding it must be simultaneously analyzed in order to obtain the vital design information and also in convection flows set up when the bounding surfaces absorb heat by solar radiation (see Chaudhary and Jain [1, 2]). This results in the heat transfer rate through the surface being proportional to the local difference in the temperature with the ambient conditions. Merkin [3] has illustrated that, in general, there are four common heating processes specifying the wall-to-ambient temperature distributions, namely, (a) constant or prescribed wall temperature; (b) constant or prescribed surface heat flux; (c) conjugate conditions, where heat is supplied through a bounding surface of finite thickness and finite heat capacity. The interface temperature is not known a priori but depends on the intrinsic properties of the system, namely the thermal conductivity of the fluid and solid; and (d) Newtonian heating, where the heat transfer rate from the bounding surface with a finite heat capacity is proportional to the local surface temperature and which is usually termed conjugate convective flow. The free convection boundary layer flow along a vertical surface in a porous medium with Newtonian heating has been presented by Lesnic et al. [4]. Excellent reviews of the topics of conjugate heat transfer problems can be found in the book edited by Kimura et al. [5].

Further, there has been an increasing interest in the flow of time-independent non-Newtonian fluids through tubes possessing a definite yield value because of their applications in polymer processing industries. The most popular among these fluids is the Casson fluid. We can define a Casson fluid as a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs and a zero viscosity at an infinite rate of shear. The Casson model is a well-known rheological model for describing the non-Newtonian flow behavior of fluids with a yield stress [6]. A Casson fluid is a type of non-Newtonian fluid. The examples of Casson fluid are of the type are as follows: jelly, tomato sauce, honey, soup, concentrated fruit juices, etc. Human blood can also be treated as Casson fluid. Due to the presence of several substances like, protein, fibrinogen, and globulin in aqueous base plasma, human red blood cells can form a chainlike structure, known as aggregates or rouleaux. The model was developed for viscous suspensions of cylindrical particles [7]. Regardless of the form or type of suspension, some fluids are particularly well described by this model because of their nonlinear yield-stress-pseudoplastic nature. Examples are blood [8], chocolate [9], xanthan gum solutions [10]. The Casson model fits the flow data better than the more general Herschel–Bulkley model [11, 12], which is a power-law formulation with yield stress [13, 14]. For chocolate and blood, the Casson model is the preferred rheological model. It seems increasingly that the Casson model fits the nonlinear behavior of yield-stress-pseudoplastic fluids rather well and it has therefore gained in popularity since its introduction in 1959. It is relatively simple to use, and it is closely related to the Bingham model [13, 14], which is very widely used to describe the flow of slurries, suspensions, sludge, and other rheologically complex fluids [15]. Eldabe and Salwa [16] have studied the Casson fluid for the flow between two rotating cylinders, and Boyd et al. [17] investigated the Casson fluid flow for the steady and oscillatory blood flow. Boundary layer flow of Casson fluid over different geometries is considered by many authors in recent years. Nadeem et al. [18] presented MHD flow of a Casson fluid over an exponentially shrinking sheet. Kumari et al. [19] analyzed peristaltic pumping of a MHD Casson fluid in an inclined channel. Sreenadh et al. [20] studied the flow of a Casson fluid through an inclined tube of non uniform cross-section with multiple stenos. Mukhopadhyay et al. [21] studied the unsteady two-dimensional flow of a non-Newtonian fluid over a stretching surface having a prescribed surface temperature, the Casson fluid model is used to characterize the non-Newtonian fluid behavior. The details of steady, fully-developed and laminar flow of Casson fluids have been described in Fung [22]. In view of the non-

Newtonian nature of blood in capillaries and the filtration/absorption property of the walls, Oka [23] studied blood flow in capillaries with permeable walls using the Casson fluid model.

Having in mind the above reported studies on the boundary layer flow due to stretching sheet, we venture further in the regime of two-dimensional flows of the Casson fluid. Casson fluid model is used to characterize the non-Newtonian fluid behavior. In addition, the fluid is taken to be electrically conducted and the flow is induced by a stretching sheet with Newtonian heating in which the heat transfer from the surface is proportional to the local surface temperature.

2. Problem Analysis

Consider the steady two-dimensional laminar flow and heat transfer of a non-Newtonian Casson fluid caused by a stretching sheet with Newtonian heating (NH) in which the heat transfer from the surface is proportional to the local surface temperature. Casson fluid model is used to characterize the non-Newtonian fluid behavior, (physical model and coordinate system is shown in Figure 1). In our analysis the x-axis and y-axis are taken as the coordinates parallel to the plate and normal to it, respectively, and the fluid occupies the region $y \geq 0$. Moreover, we assume that the wall is subjected to a Newtonian heating of the form proposed by Merkin [3]. Moreover, the rheological equation of state for an isotropic and incompressible flow of a Casson fluid as [16]

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{P_y}{\sqrt{2\pi}}\right) e_{ij}\,, & \pi > \pi_c \\ 2\left(\mu_B + \frac{P_y}{\sqrt{2\pi_c}}\right) e_{ij}\,, & \pi < \pi_c \end{cases} \tag{1}$$

where, τ_{ij} is the (i,j) -th component of the stress tensor, $\tau_{ij} = e_{ij}e_{ij}$ and e_{ij} are the (i,j) -th component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non-Newtonian fluid, and P_y is the yield stress of the fluid. So, if a shear stress less than the yield stress is applied to the fluid, it behaves like a solid, whereas if a shear stress greater than the yield stress is applied, it starts to move. Considering the balance laws of mass, linear momentum and energy and with the help of Boussinesq's approximation the equations governing this flow can be written in the usual form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

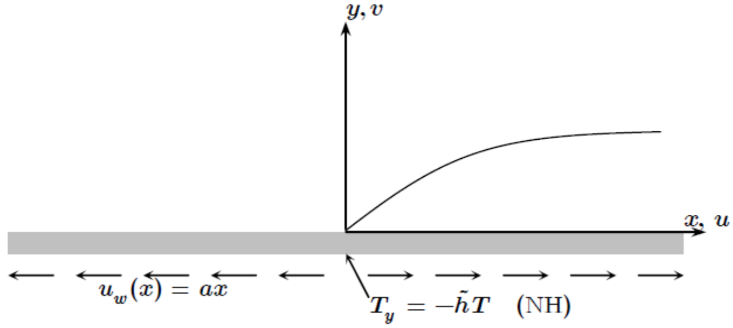

Figure 1. Physical model and coordinate system.

The appropriate boundary conditions for the governing equations are

$$y = 0, \qquad u = u_w(x) = ax, \qquad v = 0, \qquad \frac{\partial T}{\partial y} = -\tilde{h}T \quad \text{(NH)}$$
$$y \to \infty, \qquad u \to 0, \quad T \to T_\infty \tag{5}$$

where, (u, v) are the velocity components in (x, y) directions, respectively, ν is the kinematic viscosity, and α is the thermal diffusivity of the fluid. T is the temperature of the fluid inside the thermal boundary layer, whereas T_∞ is the ambient temperature, β is the Casson parameter. $u_w(x)$ is the velocity of the stretching surface a being a positive constant. $\tilde{h}$ is the heat transfer parameter.

In order to get a similarity solution of the problem we introduce the following non-dimensional variables.

$$\eta = \sqrt{\frac{a}{\nu}}\,y, \qquad \psi = \sqrt{a\nu}\,xf(\eta), \qquad \theta(\eta) = \frac{T - T_\infty}{T_\infty}\,\text{(NH)},$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad \text{(CWT)}, \qquad \theta(\eta) = \frac{k}{q_w}\sqrt{\frac{a}{\nu}}(T - T_\infty) \quad \text{(CHF)} \tag{6}$$

The similarity solution proceeds by selecting a stream function such as $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$ so that the continuity equation (2) is automatically satisfied.

Substituting Eq. (6) into Eqs. (3) and (4) we obtain the following ordinary differential equations, which are locally similar.

$$\left(\frac{1+\beta}{\beta}\right)f''' + ff''' - f'^2 = 0 \tag{7}$$

$$\frac{1}{\Pr}\theta'' + f\theta' = 0 \tag{8}$$

The boundary conditions (5) then turn into

$$f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -\gamma\big(1 + \theta(0)\big) \qquad \text{(NH)}$$
$$f(\eta \to \infty) \to 0, \qquad \theta(\eta \to \infty) \to 0 \tag{9}$$

Furthermore, considering the two cases

$\theta(0) = 1$ (CWT) and $\theta'(0) = -1$ (CHF)

The prime denotes ordinary differentiation with respect to the similarity variable, and $\gamma = \tilde{h}\sqrt{\frac{\nu}{a}}$ is the conjugate parameter for Newtonian heating and $\Pr = \nu\alpha^{-1}$ is the Prandtl number. It is worth mentioning that when $\gamma = 0$, an insulated wall is presented and when $\gamma \to \infty$, the wall temperature remains constant.

In addition, the exact analytical solution of Eq. (7) is given as

$$f(\eta) = \beta^*\left(1 - e^{-\frac{\eta}{\beta^*}}\right), \quad f'(\eta) = e^{-\frac{\eta}{\beta^*}}, \quad f''(\eta) = -\frac{1}{\beta^*}e^{-\frac{\eta}{\beta^*}} \tag{10}$$

With $\beta^* = \sqrt{1 + \frac{1}{\beta}}$, now, we can write that if $f(\eta)$ is given by the exact solution (10), then the temperature distribution $\theta(\eta)$ can be solved analytically as

$$\theta(\eta) = C_1 \int_\eta^\infty e^{-\Pr\int_\eta^\infty f d\eta}\, d\eta$$

with

$$C_1 = -\frac{(1+\theta(0))}{e^{-\Pr\int_0^\infty f d\eta}} \qquad \text{(for NH)}$$
$$C_1 = \frac{1}{\int_0^\infty e^{\left\{-\Pr\int_\eta^\infty f d\eta\right\}} d\eta} \qquad \text{(for CWT)} \qquad (11)$$
$$C_1 = -\frac{1}{e^{\left\{-\Pr\int_0^\infty f d\eta\right\}}} \qquad \text{(for CHF)}$$

In practical applications, the quantity of physical interest in our case is the local Nusselt number Nu, which can be written in non-dimensional form as

$$Nu = \frac{x q_w}{k(T_w - T_\infty)}$$

Here q_w, is the surface heat flux and defined as $q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$ and k is the thermal conductivity. Considering the similarity variables (6) we may obtain

$$\frac{Nu}{\sqrt{Re}} = 1 + \frac{1}{\theta_w} \qquad \text{(for NH), or}$$
$$\frac{Nu}{\sqrt{Re}} = -\theta_w' \qquad \text{(for CWT), or}$$
$$\frac{Nu}{\sqrt{Re}} = \frac{1}{\theta_w} \qquad \text{(for CHF)}$$

where $Re = ax^2\nu^{-1}$ is the local Reynolds number.

3. Results and discussion

Numerical calculations are obtained for the problem of boundary layer flow heat transfer of Casson fluid past a flat plate with Newtonian heating (NH) in which the heat transfer from the surface is proportional to the local surface temperature. The set of the coupled Equations (7) and (8) is highly nonlinear and cannot be solved analytically. Together with the boundary conditions (9) they form a two-point boundary value problem (BVP) which can be solved using the routine bvpc45 of the symbolic computer algebra software MATLAB by converting it into an initial value problem (IVP). In this way we have to choose a finite value of the boundary $\eta \to \infty$, say h_{finite}. Care has been taken in choosing h_{finite} for a given set of parameters because for a fixed value of h_{finite} for all calculations may produce inaccurate results. The results are given to carry out a parametric study showing influences of several non-dimensional parameters, namely, Casson parameterβ, Prandtl number Pr and conjugate parameter γ. For the validation of the numerical results obtained in this study, the case when the Casson parameter is absent has been considered and compared with the previously published results. Tables 1-3 present the numerical values of temperature at the wall $\theta(0)$ (for NH, CWT and CHF cases) and temperature gradient $-\theta'(0)$ (for CWT case) along with the results reported by Hassanien et al. [24], Ishak et al. [25], Salleh et al. [26] and Elbashbeshy [27], which show an excellent agreement. Considering Newtonian fluid (i.e. $\beta \to \infty$), it is noticed that for the case of CWT, as Pr increases, the both values of $\theta(0)$ and $-\theta'(0)$ increase. Moreover, for the case of CHF, the wall temperature $\theta(0)$ decreases as Pr increases. However, for the case of NH, both $\theta(0)$ and $-\theta'(0)$ decrease as Pr increases. On the other hand, it is observed that for the case of NH, for small Prandtl number, the decrease in $\theta(0)$ and $-\theta^{'}(0)$ is very significant with increasing Prandtl number. The trend for NH case is similar to the CHF case but different from the CWT case. Table 3 presents the values of Nusselt number for CWT, CHF and NH cases with various values of Pr. From this table, it is found that the values of the local Nusselt number are very well comparable with the results reported by Hassanien et al. [24] and Ishak et al. [25] (for CWT case) and also Elbashbeshy [27] and Salleh et al. [26] (for CHF case). In addition, Table 4 reported the local Nusselt number for different values of Casson parameter and it is clear that an increase in β tends to decrease local Nusselt number for the two cases CHF and NH.

Table 1. Comparison values of $\theta(0)$ and $-\theta'(0)$ for various values of Pr when $\gamma = 1$ (NH), as $\beta \to \infty$.

Pr	$\theta(0)$		$-\theta^{'}(0)$	
	Salleh et al. [26]	Present	Salleh et al. [26]	Present
5	1.76594	1.773267	2.76594	2.78326
7	1.13511	1.130948	2.13511	2.12794
10	0.76531	0.764490	1.76531	1.76449
100	0.16115	0.147803	1.16115	1.14780

Table 2. Comparison values of $Nu/\sqrt{Re}$ for CWT, CHF and NH cases with different values of Pr when $\gamma = 1$ (NH), as $\beta \to \infty$.

Pr	$-\theta^{'}(0)$ (CWT)				$\theta(0)$ (CHF)			
	Exact Eq. (11)	Hassanien et al. [24]	Salleh et al. [26]	Present	Exact Eq. (11)	Hassanien et al. [24]	Salleh et al. [26]	Present
0.72		0.46325	0.46317	0.46317		2.13767	2.15902	2.15512
1.0	0.58202	0.58198	0.58198	0.58198	1.71816	1.71792	1.71828	1.71801
3.0	1.16525	1.16525	1.16522	1.16524	0.85819		0.85817	0.85818
5.0	1.56805		1.56806	1.56801	0.63773		0.63770	0.63770
7.0	1.89540		1.89548	1.89539	0.52759		0.52755	0.52757
10.0	2.30800	2.30801	2.30821	2.30801	0.43327	0.43341	0.43322	0.43325
100.0	7.76565	7.74925	7.76249	7.76233	0.12877		0.12851	0.12862

Table 3. Comparison values of $Nu/\sqrt{Re}$ for CWT, CHF and NH cases with different values of Pr when $\gamma = 1$ (NH), as $\beta \to \infty$.

Pr	$-\theta^{'}(0)$ (CWT)			$1/\theta(0)$ (CHF)			$1 + (1/\theta(0))$ (NH)	
	Ishak et al. [25]	Hassanien et al. [24]	Present	Elbashbeshy [27]	Salleh et al. [26]	Present	Salleh et al. [26]	Present
0.72			0.46314	0.46780	0.46317	0.46314		
1.0	0.5820	0.58198	0.58197	0.5820	0.58210	0.58197		
3.0	1.1652	1.16522	1.16524		1.16527	1.16524	1.16595	1.16581
10.0	2.3080	2.30821	2.30804	2.3080	2.30728	2.30801	2.30666	2.30806

Figures 2 and 3 depict the effects of Casson parameter β on temperature and velocity, distributions for Casson fluid and Newtonian fluid ($\beta \to \infty$), respectively considering the case of Newtonian heating (NH). The increasing values of the Casson parameter i.e. the decreasing yield stress (the fluid behaves as Newtonian fluid as Casson parameter becomes large) suppress the velocity field. The effect of increasing values of β is to reduce the rate of transport, and hence, the boundary layer thickness decreases. It is observed that $f'(\eta)$ and the associated boundary layer thickness are decreasing function of β. The effect of increasing β leads to enhance the temperature profile. The thickening of the thermal boundary layer occurs due to increase in the elasticity stress parameter.

Influence of the conjugate parameter on heat transfer distribution for the case of Newtonian heating (NH) is shown in Figure 4. As it is observed increasing in conjugate parameter γ tends to increase the temperature distribution. Moreover, for the case of Newtonian heating (NH) increasing conjugate parameter γ tends to increase the temperature at surface $\theta(0)$ and absolute value of $\theta'(0)$, as observed from Figures 8 and 9. On the other hand, the behavior of the temperature distributions for the variation of Prandtl number considering the cases of (NH) with $\gamma = 1$ is illustrated in Figure 5. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. It is seen that the temperature decreases with increasing Pr. Furthermore, the thermal boundary layer thickness decreases sharply by increasing Prandtl number. The temperature gradient at surface is negative for all values of Prandtl number as seen from Figure 6, which means that the heat is always transferred from the surface to the ambient fluid. Fluids with lower Prandtl number will possess higher thermal conductivities (and thicker thermal boundary layer structures), so that heat can diffuse from the surface faster than for higher Pr fluids (thinner boundary layers). Physically, if Pr increases, the thermal diffusivity decreases and this phenomenon leads to the decreasing of energy transfer ability that reduces the thermal boundary layer. In addition, Figure 7 displays that an increase in temperature at the surface occurs with increasing Prandtl number.

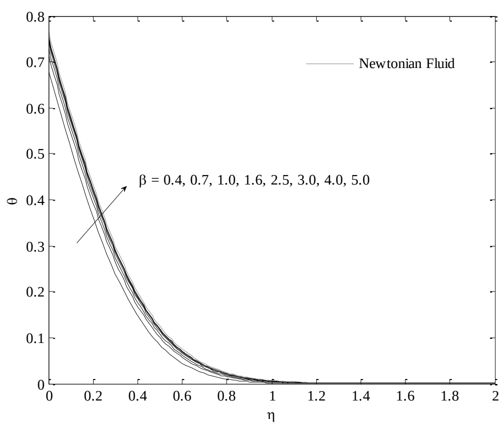

Figure 2. Variation of the temperature for various Casson parameter when $\gamma = 1, Pr = 10$.

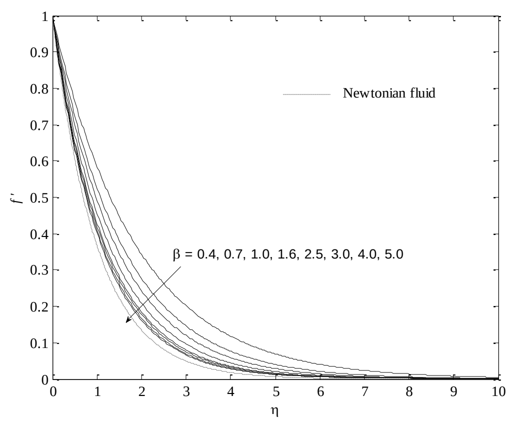

Figure 3. Variation of the velocity for various Casson parameter when $\gamma = 1, Pr = 10$.

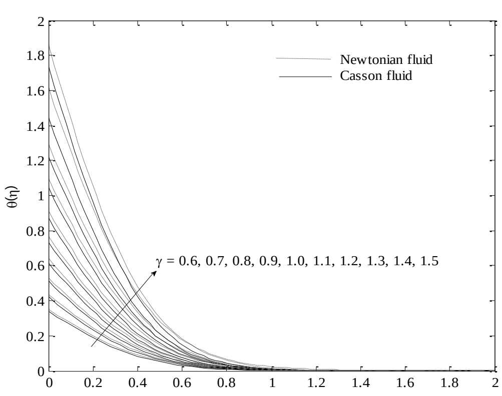

Figure 4. Variation of the temperature for various conjugate parameter when $Pr = 10, \beta = 1.2, \beta \to \infty$.

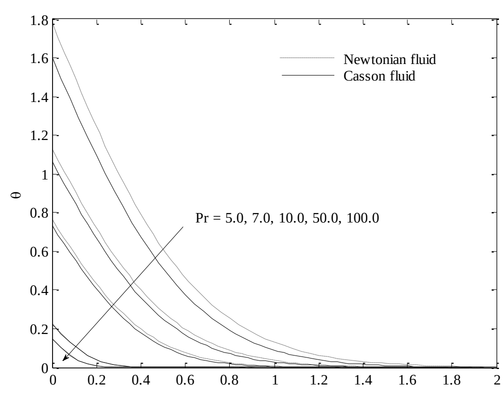

Figure 5 Variation of the temperature for various Prandtl number when $\gamma = 1, \beta = 1.2, \beta \to \infty$.

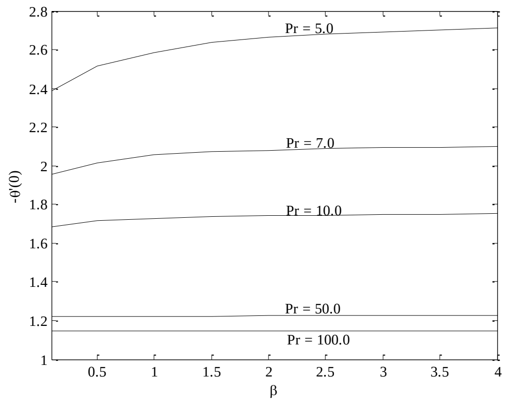

Figure 6. Variation of the wall temperature gradient with Casson parameter when $\gamma = 1$ for various Prandtl number.

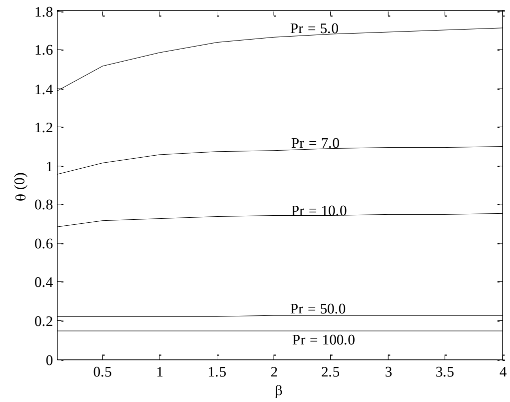

Figure 7. Variation of the wall temperature with Casson parameter when $\gamma = 1$ for various Prandtl number.

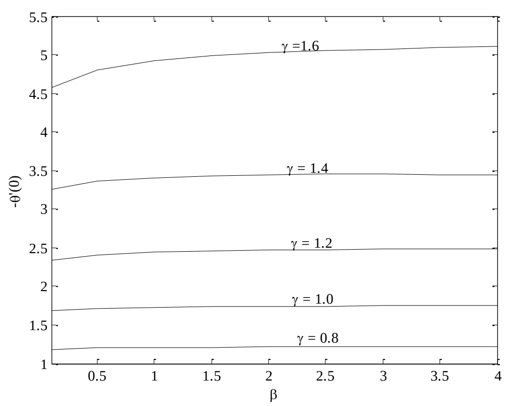

Figure 8. Variation of the wall temperature gradient with Casson parameter when $\gamma = 1$ for various conjugate parameter.

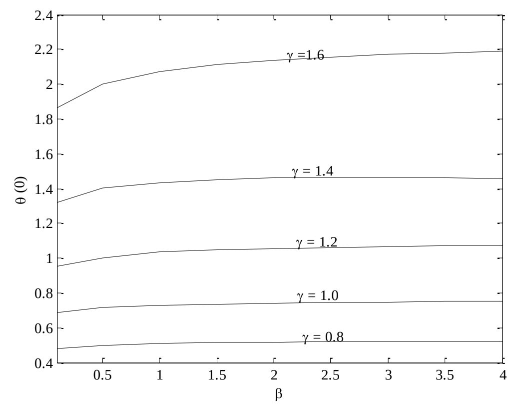

Figure 9. Variation of the wall temperature with Casson parameter when $Pr = 10$ for various conjugate parameter.

4. Conclusions

A numerical solution is carried out to analyze the problem of steady, two dimension boundary layer flow and heat transfer of non-Newtonian Casson fluid over a stretching sheet with Newtonian heating. It is shown in this paper how the Prandtl number Pr, conjugate parameter and Casson parameter affect the temperature distribution, the wall temperature and the heat transfer coefficient. We can conclude that (for the case of NH):

■ The thermal boundary layer thickness depends on the conjugate parameter (Newtonian heating) γ . Moreover; it is found that an increase in γ results an increase in the temperature distribution.

■ The effect of increasing values of the Casson parameter β is to suppress the velocity field, whereas the temperature is enhanced with increasing Casson parameter. Further Nusselt number decreases for the case of NH with increasing Casson parameter.

■ The thermal boundary layer thickness depends strongly on the Prandtl number Pr. Further, it is found that an increase in Pr results in a decrease of the temperature distribution.

Table 4. Values of $Nu\ /\ \mathrm{Re}^{1/2}$ for CHF and NH cases with different values of β when $\gamma = 1$.

β	Pr	$1/\theta(0)$ (CHF)	$1 + (1/\theta(0))$ (NH)
0.05	5	3.474472	1.73532
	7		2.06228
	10		2.47447
0.5	5	3.39950	1.66009
	7		1.98718
	10		2.39950
1.0	5	3.37154	1.63197
	7		1.95913
	10		2.37153
2.0	5	3.34786	1.60814
	7		1.93538
	10		2.34786
3.0	5	3.33712	1.59734
	7		1.92461
	10		2.33711
4.0	5	2.91842	1.59113
	7		1.91842
	10		2.33095
5.0	5	2.91440	1.58710
	7		1.91440
	10		2.32694

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