

Effect of Reynold's Number for Mixed Convection Flow of Nanofluid in a Double Lid Driven Cavity with Heat Generating Obstacle

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Abstract

Based on heatline approach, heat transfer characteristics of nanofluid inside a double lid driven cavity with heat generating obstacle is analyzed numerically. The operational nanofluid is chosen as water-alumina nanofluid. The governing differential equations of mass, momentum and energy are solved by using Galerkin weighted residual finite element method. The behavior of the fluid in the ranges of Reynolds number Re (20 - 150) is portrayed. Numerical results are presented in terms of isotherms, stream lines, heat lines, average Nusselt number and average temperature. Results demonstrate that both the flow and thermal fields are strongly influenced by the mentioned parameter.

Keywords: *Heatline; solid volume fraction; nanofluid; Reynold's number; lid driven cavity*

Nomenclature

| | | | |
|------------|--|----------------------|--------------------------------------|
| c_p | specific heat at constant pressure | X, Y | dimensionless Cartesian coordinates |
| g | gravitational acceleration | Greek symbols | |
| k | thermal conductivity of fluid | α | thermal diffusivity |
| L | length of the cavity | β | coefficient of thermal expansion |
| N | dimensionless distance either along X or Y direction | θ | dimensionless fluid temperature |
| Nu | average Nusselt number | θ_s | dimensionless solid temperature |
| p | pressure | $\Delta\theta$ | dimensionless temperature difference |
| P | non dimensional pressure | μ | dynamic viscosity of the fluid |
| Pr | Prandtl number | ν | kinematic viscosity |
| q | heat flux | ρ | density of the fluid |
| Re | Reynolds number | ψ | stream function |
| Ri | Richardson number | Π | dimensionless heat function |
| T | dimensional fluid temperature | X | solid volume fraction |
| T_s | dimensional solid temperature | Subscripts | |
| ΔT | dimensional temperature difference | av | average |
| u, v | velocity components | c | cylinder center |
| U, V | dimensionless velocity | f | fluid |
| U_o | lid velocity | h | heated wall |
| \bar{V} | cavity volume | nf | nanofluid |
| x, y | Cartesian coordinates | s | solid |

1. Introduction

Mixed convection flows occur when each of forced convection and natural convection dominates the other. The study of mixed convection in lid-driven enclosures has received a continuous attention, due to the interest of the phenomenon in many technological processes. These include design of solar collectors, thermal design of buildings, air conditioning and, recently the cooling of electronic circuit boards. The present study simulates a reasonable system such as air-cooled electronic equipment with a heat component or an oven with heater.

Cooling mechanism is one of the most important concerns in the industry. In most cases cooling improvement of existing heat exchanger is done by increasing the size of these systems which is not desirable. In recent years nanofluid is used to improve the heat transfer since it is with higher thermal conductivity than the pure fluid.

Rahman et al. [1] investigated the effect of Reynolds and Prandtl numbers effects on MHD mixed convection in a lid-driven cavity along with joule heating and a centered heat conducting circular block. Heat transfer and flow characteristics for MHD mixed convection in a lid driven cavity with heat generating obstacle is observed by Billah et al. [2]. The authors claimed that only the thermal fields are affected by thermal conductivity ratio K in the cavity at all convective regimes. A review on the subject shows that a sizeable number of authors had considered MHD mixed convection in enclosures Parvin and Hossain and Parvin et al. [3, 4]. Rahman et al. [5] made a numerical analysis of Effect of heat-generating solid body on mixed convection flow in a ventilated cavity. Numerical simulation of mixed convection within nanofluid filled cavities with two adjacent moving walls is analyzed by Esfe et al. [6]. Again Saedodin et al. [7] presented mixed convection heat transfer performance in a ventilated inclined cavity containing heated blocks. A nanofluid is a fluid containing nanometer sized particles, called nanoparticles. Muthamilsevan and Doh [8] examined mixed convection of heat generating nanofluid in a lid driven cavity with uniform and non-uniform heating of bottom wall. The results show that when the moving lids have opposite effect, the streamlines contain two main vortices. Again heatline analysis for MHD mixed convection flow of nanofluid in a driven cavity with heat generating block was studied by Parvin and Siddiqua [9]. An enhancement in heat transfer rate is observed with the increase of nanoparticles volume fraction. Effect of solid volume fraction and tilt angle in a quarter circular solar thermal collectors filled with CNT–water nanofluid was investigated by Rahman et al. [10]. Numerical analysis on mixed convection heat transfer of nanofluid in a channel is investigated by Rashidi et al. [11]. Recently, Kandasmy et al. [12] performed Nanoparticle volume fraction with heat and mass transfer on MHD mixed convective flow in a nanofluid in presence of thermo-diffusion under convective boundary condition. The heatline concept was first introduced by Kimuru and Bejan and Bejan [13, 14]. Heatline represents heat flux lines which represent the trajectory of heat flow and they are normal to the isotherms for conductive heat transfer. Basak et al. [15, 16] also investigated the heatline approach on natural and mixed convection.

To the best knowledge of the authors' knowledge, a little attention has been paid to problem of mixed convection in a double lid driven cavity filled with water alumina nanofluid with a square heat generating obstacle. The present work focuses on the heatline analysis for Reynolds number effect on mixed convection flow in a double lid driven cavity. It is expected that the present numerical investigation will contribute to the search of finding more efficient and better renewable energy equipment.

2. Physical and Mathematical Model

The schematic of the problem herein investigated is presented in Figure 1(a)-(b). A Cartesian coordinate system is used with the origin at the lower left corner of the computational domain. The system consists of a double-lid-driven square enclosure with sides of length L . A heat generating solid square obstacle is positioned at the center of the cavity. In addition, the cavity is saturated with water- Al_2O_3 nanofluid. The solid obstacle has a thermal conductivity of k_s and generates uniform heat flux (q) per unit area. Moreover, the vertical walls of the cavity are mechanically lid-driven and considered to be at a constant temperature T_c and uniform velocity V_0 in the positive y direction (upward). Furthermore, both the top and bottom surfaces of the enclosure are kept adiabatic. Due to the symmetry of the domain, half of the domain is considered for the computation.

For a steady, two-dimensional laminar and incompressible flow, the governing equations may be written in the non-dimensional form as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\gamma_{nf}}{\gamma_f} \frac{1}{Re} \nabla^2 U \quad (2)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\gamma_{nf}}{\gamma_f} \nabla^2 V + Ri \frac{\beta_{nf}}{\beta_f} \theta \quad (3)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Re Pr} \frac{\alpha_{nf}}{\alpha_f} \nabla^2 \theta \quad (4)$$

For solid obstacle the energy equation is

$$\nabla^2 \theta_s + Q = 0 \quad (5)$$

where $Re = \frac{V_0 L}{\nu}$, $Pr = \frac{\nu}{\alpha}$, $Ri = \frac{g \beta \Delta T L}{V_0^2}$, $\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$, $\rho_{nf} = \chi \rho_s + (1 - \chi) \rho_f$,

$(\rho c_p)_{nf} = \chi (\rho c_p)_s + (1 - \chi) (\rho c_p)_f$, $(\rho \beta)_{nf} = \chi (\rho \beta)_s + (1 - \chi) (\rho \beta)_f$, $\mu_{nf} = \frac{\mu_f}{(1 - \chi)^{2.5}}$,

$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\chi(k_f - k_s)}{k_s + 2k_f + 2\chi(k_f - k_s)}$ and $Q = \frac{q L^2}{k_s \Delta T}$ are Reynold's number, Prandtl's number, Richardson's number, thermal diffusivity, effective density, heat capacitance, thermal expansion coefficient, effective viscosity, effective thermal conductivity and heat generating parameter in the solid, respectively.

Eq. (1)-(5) are non-dimensionalized by using the following dimensionless parameters:

$$X = \frac{x}{L}, Y = \frac{y}{L}, V = \frac{v}{U_0}, U = \frac{u}{U_0}, \Delta T = T_h - T_c, \theta = \frac{T - T_c}{\Delta T}, P = \frac{p}{\rho_{nf} U_0^2}, \theta_s = \frac{T_s - T_c}{\Delta T}$$

The boundary conditions for the present problem are specified as follows:

At sliding double leads: $U = 0, V = 1, \theta = 0$

At horizontal top and bottom walls: $U = V = 0, \frac{\partial \theta}{\partial N} = 0$

At square block boundaries: $U = V = 0, \theta = \theta_b$

At fluid solid interface: $(\frac{\partial \theta}{\partial N})_{fluid} = K (\frac{\partial \theta_s}{\partial N})_{solid}$

Where N is the non-dimensional distances either in X or Y direction acting normal to the surface and K is the ratio of the solid fluid thermal conductivity K_s/K_f .

The relationships between stream function ψ and velocity components U, V for two-dimensional flows are

$$U = \frac{\partial \psi}{\partial Y} \text{ and } V = -\frac{\partial \psi}{\partial X}, \text{ which give a single equation}$$

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X}$$

The no-slip condition is valid at all boundaries as there is no cross-flow. Hence $\psi = 0$ is used for boundaries.

The heat flow within the enclosure is displayed using the heat function Π obtained from conductive heat fluxes $\left(-\frac{\partial \theta}{\partial X}, -\frac{\partial \theta}{\partial Y}\right)$ as well as convective heat fluxes $(U\theta, V\theta)$. The heat function satisfies the

steady energy balance equation $\frac{\partial \Pi}{\partial Y} = U\theta - \frac{\partial \theta}{\partial X}$ and $-\frac{\partial \Pi}{\partial X} = V\theta - \frac{\partial \theta}{\partial Y}$ which yield a single equation

$$\frac{\partial^2 \Pi}{\partial X^2} + \frac{\partial^2 \Pi}{\partial Y^2} = \frac{\partial}{\partial Y}(U\theta) - \frac{\partial}{\partial X}(V\theta)$$

The average Nusselt number, average temperature and average velocity may be expressed as

$$Nu = -\frac{1}{S} \int_0^S \left(\frac{k_{nf}}{k_f}\right) \frac{\partial \theta}{\partial N} dN \quad \theta_{av} = \int \theta d\bar{V} / \bar{V} \text{ and } v_{av} = \int v d\bar{V} / \bar{V} \text{ respectively, where } S \text{ is the non-}$$

dimensional length of the surface and \bar{V} is the volume to be accounted.

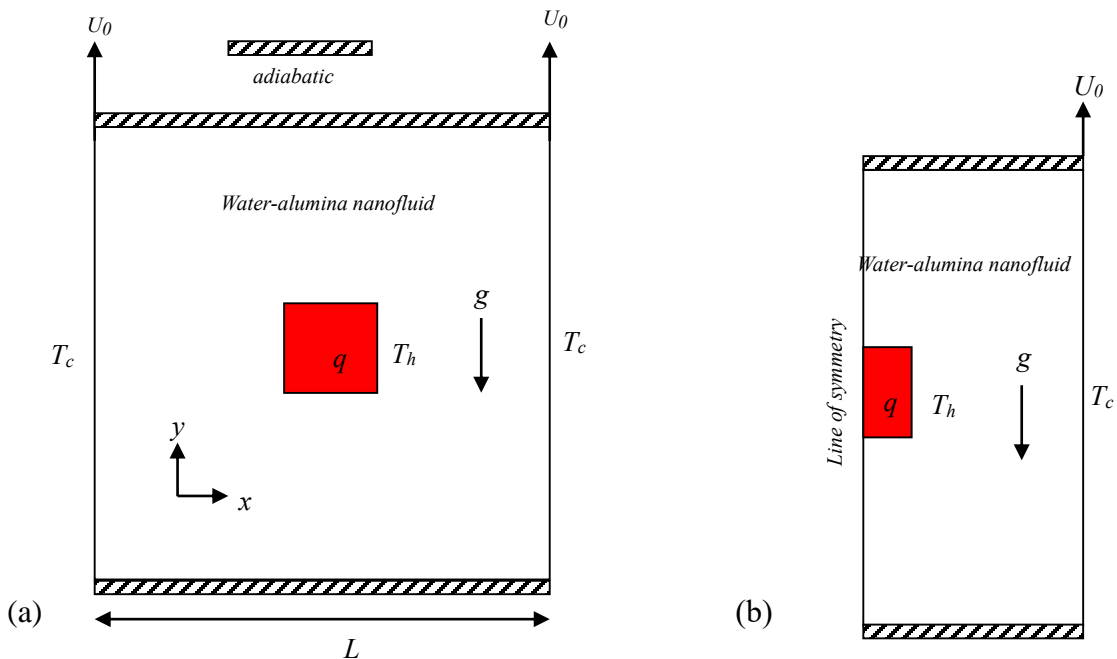


Figure 1. (a) Schematic diagram of the problem and (b) computational domain

3. Computational Procedure

The governing equations have been solved by using the Galerking weighted residual finite element method. The fundamental unknowns for the governing equations are the velocity components (U, V), the temperature θ and the pressure P . The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion ε such that $\psi^{n+1} - \psi^n \leq 10^{-4}$, where n is the number of iteration and Ψ is a function of U, V and θ .

An extensive mesh testing procedure is conducted to guarantee a grid-independent solution for $Ri = 1$, $Re = 100$, $Pr = 6.2$, $K = 5$ and $Q = 1$ and $\chi = 0.05$ in the considered domain. Five different non-uniform grid systems with the following number of elements within the resolution field: 1683, 2535, 4369, 10843 and 26638 are examined. The numerical scheme is carried out for highly precise key in the average Nusselt (Nu) number to understand the grid fineness as shown in Figure 2. The scale of the average Nusselt numbers for 10843 elements shows a little difference with the results obtained for the other elements. Hence, considering the non-uniform grid system of 10843 elements is preferred for the computation.

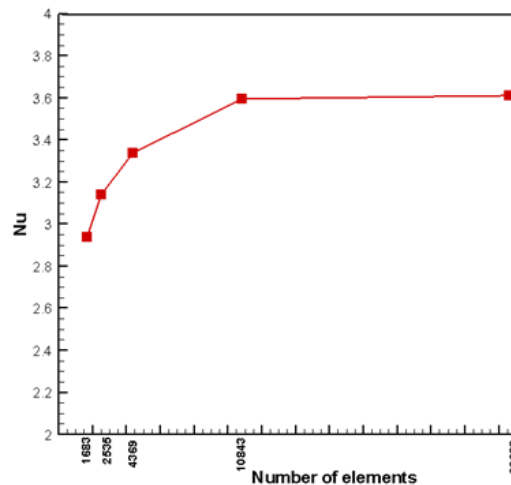


Figure 2. Grid sensitivity test

4. Results and Discussion

Mixed convection flow for nanofluid inside a lid driven cavity having a heat generating square obstacle is governed by different parameter namely heat generation Q , solid volume thermal conductivity ratio K , Richardson's number Ri , Reynold's number Re , Prandtl's number Pr . Here the Reynolds number Re is involved to control the heat transfer and fluid flow in this study. The other parameters are kept fixed at $Ri = 1$, $\chi = 0.05$, $Pr = 6.2$, $K = 5$ and $Q = 1$. The results are presented in terms of isotherms, streamlines and heatlines pattern.

The isotherms for various Reynolds number inside a double lid driven cavity with a square heat generating block are shown in Figure 3. From the figure it can be said that the Reynolds number effects significantly on isotherm structure. Isothermal lines become more condensed in the fluid near the heat generating obstacle for increasing values of Re which indicates higher temperature gradient. A thermal plume is developed near the vertical wall for higher values of Re .

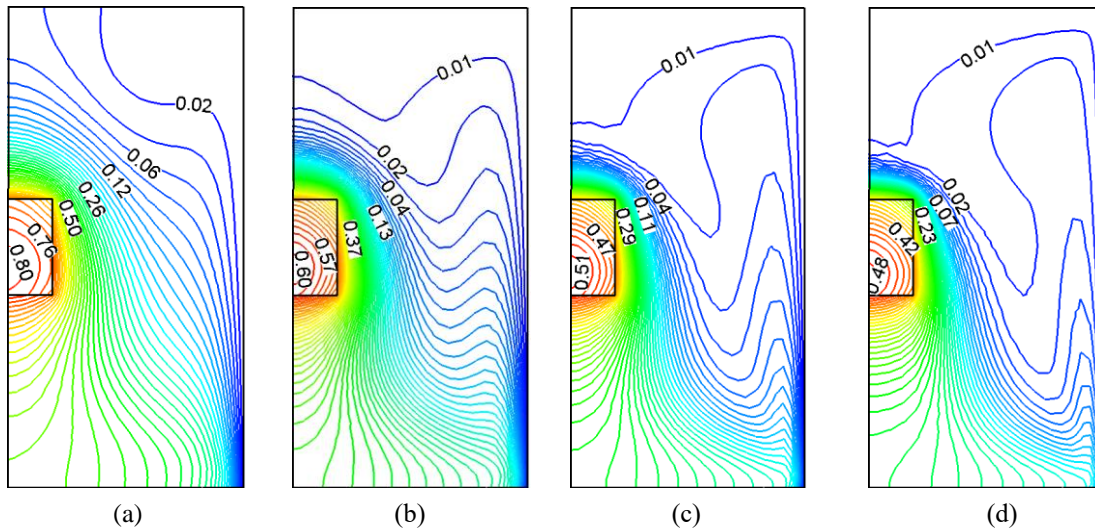


Figure 3. Isotherms for (a) $Re=20$, (b) $Re = 50$, (c) $Re = 100$ and (d) $Re = 150$ with $Ri=1$, $\chi = 5\%$, $Pr = 6.2$, $K = 5$ and $Q = 1$.

The flow field inside a double lid driven cavity with heat generating object in terms of computed streamlines for various Reynolds number is shown in Figure 4. Here the arrows indicate the direction of the streamlines and the flow field. The size of the vortex as well as the flow strength has a small effect as the Re is increased from 20 to 150. For the given boundary condition there forms a counter clockwise vortex inside the cavity. The elliptic shape of the core of the vortex becomes circular as Reynolds number gets the higher values. If the Reynolds number is increased the vortex inside the cavity becomes slightly stronger.

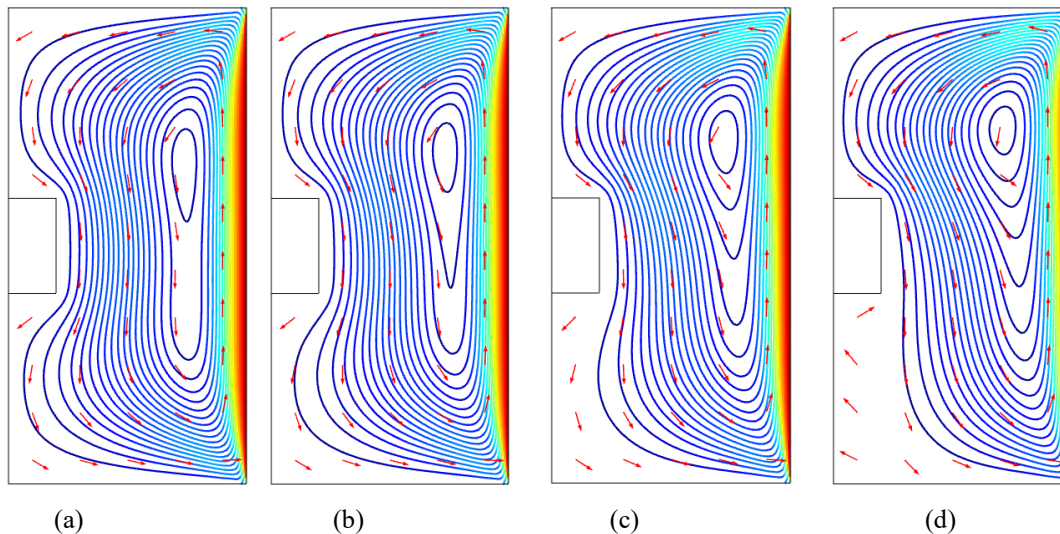


Figure 4. Streamlines for (a) $Re = 20$, (b) $Re = 50$, (c) $Re = 100$ and (d) $Re = 150$ with $Ri=1$, $\chi = 5\%$, $Pr = 6.2$, $K = 5$ and $Q = 1$.

The heat lines for various Reynolds number are shown in Figure 5. From the figure it can easily be said that there is well-built effect of Re on heat line structure. Heat flux lines originated from

the solid block becomes denser for higher values of Re . If Re increases, then the heat flow increases because of stronger inertia force.

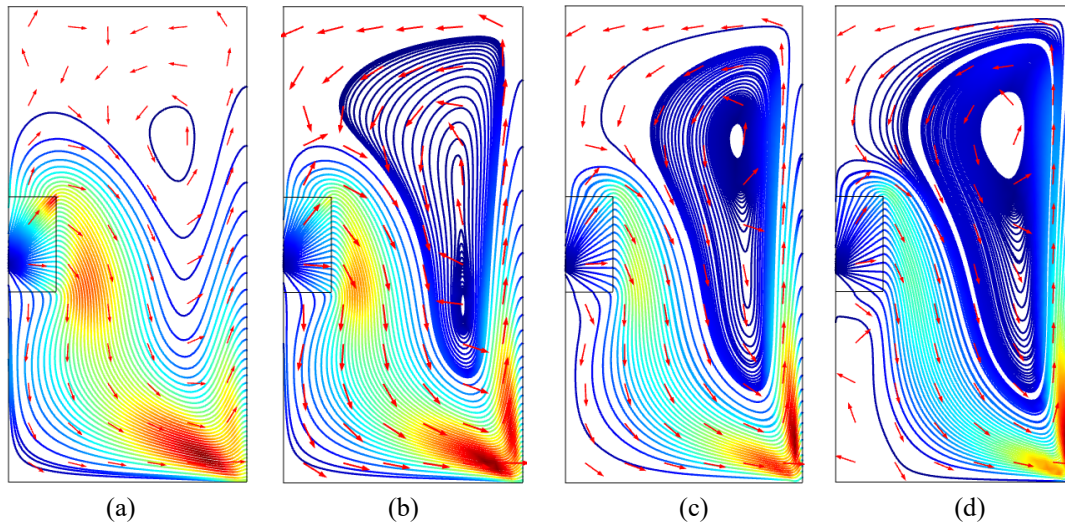


Figure 5. Heatlines for (a) $Re = 20$, (b) $Re = 50$, (c) $Re = 100$ and (d) $Re = 150$ with $Ri = 1$, $\chi = 5\%$, $Pr = 6.2$, $K = 5$ and $Q = 1$.

As shown in Figure 6(a), average Nusselt number is calculated at the heat generating obstacle. It is found from the figure that average Nusselt number increases almost linearly if Re increases. It is notable from Figure 6(b) that the average temperature in the solid and in the fluid both decreases. Decrement of temperature is more in the fluid than the solid. But in case of velocity it can be said that if the solid volume fraction increases then average velocity decreases slowly.

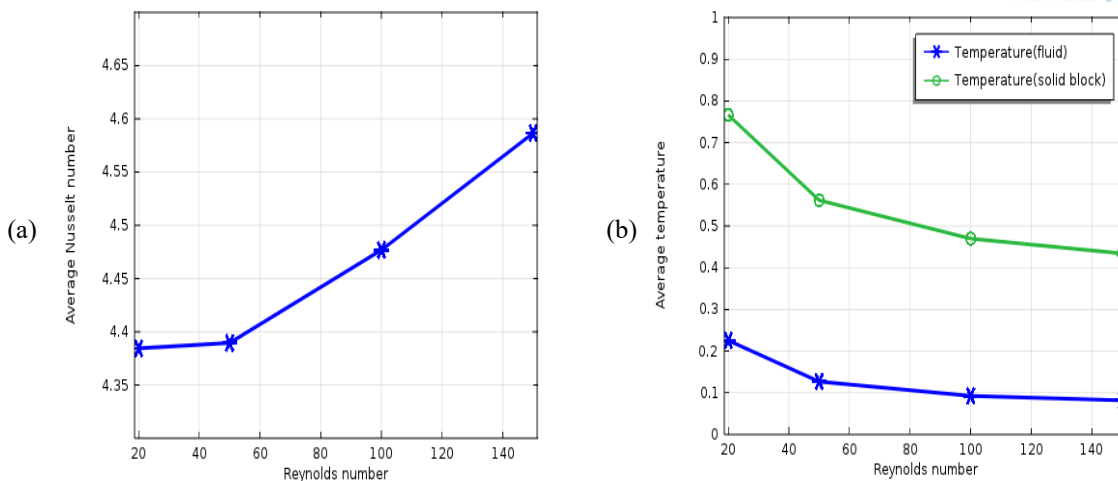


Figure 6. (a) average Nusselt number at the solid body, (b) average Temperature in the fluid and in the solid body for various Reynolds number with $Ri = 1$, $\chi = 5\%$, $Pr = 6.2$, $K = 5$ and $Q = 1$

5. Conclusions

A numerical simulation is performed to investigate the heatlines for mixed convection of water

alumina nanofluid in a double lid driven enclosure with a heat generating block. The following conclusion may be drawn:

- The Reynolds number effects significantly on the isotherms, streamlines and heatline structure.
- If the Re increases then the parabolic shaped isotherms become denser near the moving wall, the vortex inside the cavity becomes slightly stronger and the heat flow increases.
- The average Nusselt number increases due to the increment of Re while average temperature both in the fluid and in the solid decreases.

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